DYNSTALL: Subroutine Package with a Dynamic stall model

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Summary

A subroutine package, called DYNSTALL, for the calculation of 2D unsteady airfoil aerodynamics is described.

The subroutines are written in FORTRAN.

DYNSTALL is a basically an implementation of the Beddoes-Leishman dynamic stall model. This model is a semi-empirical model for dynamic stall. It includes, however, also models for attached flow unsteady aerodynamics. It is complete in the sense that it treats attached flow as well as separated flow. Semi-empirical means that the model relies on empirically determined constants. Semi because the constants are constants in equations with some physical interpretation. It requires the input of 2D airfoil aerodynamic data via tables as function of angle of attack.

The method is intended for use in an aeroelastic code with the aerodynamics solved by blade/element method.

DYNSTALL was written to work for any 2D angles of attack relative to the airfoil, e.g. flow from the rear of an airfoil.
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Symbols and notations

\( c \)  
Chord

\( c_v \)  
Vortex feed (equation (2.2.3) and (3.3.3.1))

\( C_{N\alpha} \)  
Lift curve slope. [1/rad] Same as \( C_{L\alpha} \)

\( C_{L\alpha} \)  
Lift curve slope. [1/rad] Same as \( C_{N\alpha} \)

\( C_L \)  
Lift coefficient

\( C_D \)  
Drag coefficient

\( C_M \)  
Moment coefficient

\( C_T \)  
Tangential force coefficient.

\( C_N \)  
Normal force coefficient.

Note that capital subscripts for lift, drag, pitching moment, tangential and normal force coefficients are used even though it refers to two-dimensional quantities. Capital letters for the subscripts is not to mix with other indices, e.g. \( n \) for time step number

\( f \), \( f' \)  
Separation point location

\( f \)  
Index for blade number IB

\( IR \)  
Index for radial element IR

\( q \)  
Pitch rate [rad/s]

\( u_p \)  
Plunge velocity (equation 3.2.2.7)

\( V_{rel} \)  
Velocity relative to two-dimensional airfoil element.

\( w \)  
Normal component of velocity relative to the airfoil (equation (3.2.2.2))

\( s \)  
Dimensionless time, Dimensionless with half-chord and \( V_{rel} \)

\( t \)  
Time

\( \alpha \)  
Angle of attack

\( \alpha_E \)  
Effective angle of attack (equation (2.1.6) and equation (3.2.2.5))

\( \alpha_i \)  
Shed wake induced angle of attack (equation (2.1.5))
\( \alpha_g \) Geometric angle of attack (equation (3.2.1.1))

\( \alpha_{75} \) \( \frac{3}{4} \) chord angle of attack (equation (3.2.1.2))

\( \Gamma \) Circulation \([m^2/s]\)

\( T_p \) Time constant for leading edge pressure lag (equation (2.3.1))

\( T_f \) Time constant for separation point position lag (equation (2.2.2))

\( T_v \) Time constant for vortex lift decay (equation (2.2.4))

2D Two-dimensional

3D Three-dimensional

**Subscripts**

\( q_s \) “quasi steady” values

\( n \) time step \( n \)

\( \infty \) Free stream conditions or conditions at infinity

\( I \) Impulsive loads

\( c \) Circulatory loads

\( pot \) Potential loads (sum of impulsive and circulatory loads)

\( v \) Vortex part of lift, drag et.c.

\( f \) Related to separation point location

\( stat \) Static (or quasi-static) conditions

\( dyn \) Unsteady conditions
1 Introduction

This report describes the unsteady profile aerodynamics model used in the blade element/momentum (BEM) code AERFORCE [1].

Unsteady aerodynamics is of great importance for wind turbines. Both for flow in the attached flow angle of attack region and for flow in the separated flow region.

Most attention is often given to the unsteady separated flow affects, commonly named “dynamic stall”. The stalling behavior of an airfoil will be quite different from its steady stalling behavior if the angle of attack is changed fast enough. The maximum lift coefficient can e.g. significantly exceed its static value. For wind turbines, dynamic stall also largely affects the aerodynamic damping in stall. Generally the damping would be largely under-predicted if steady stall data were used in aeroelastic calculations. Also, for yawed turbines, dynamic stalling plays a great role with increased $C_{l,max}$ and different phase of the aerodynamic forces as a function of rotor azimuth angle relative to using steady data. Many papers and reports have been written dealing with dynamic stall for wind turbines. A recent paper is [2] in which further references can found. Dynamic stall has also been subject of two EC research projects [3] and [4].

Unsteady aerodynamics is, however, also important for attached flow. Unsteady aerodynamics effects are important e.g. for stability calculations at turbine over-speeding, for blade-tower interactions and for fast pitching action.

The model in the DYNSTALL-package is an implementation of the Beddoes-Leishman dynamic stall model described in e.g. [5], [6] and [7]. This model is a 2D semi-empirical model for dynamic stall. It includes, however, also models for attached flow unsteady aerodynamics. It is thus complete in the sense that it treats attached flow as well as separated flow.
1.1 Structure of the report

The Beddoes-Leishman model in its original version is presented in chapter 2. Changes to this model that are introduced in the FFA DYNSTALL version are described in chapter 3.

In chapter 4 details in the Fortran implementation are given.

Finally, some recommendations on the use are given in chapter 5.
2 The Beddoes-Leishman model

The Beddoes-Leishman model is a semi-empirical model. Semi-empirical means that the model relies on empirically determined constants. Semi because the constants are constants in equations with some physical interpretation.

The model is based on tables of steady aerodynamic data ($C_L(\alpha)$, $C_D(\alpha)$ and $C_M(\alpha)$).

The unsteady behavior is described by different processes as e.g. the shed wake effect on the inflow and a dynamic delay in the separation process. The different dynamic processes are modeled as differential equations that need user-set values for constants to get best agreement with experiments or theoretical models.

The Beddoes-Leishman model can be described as an indicial response model for attached flow extended with models for separated flow effects and vortex lift. The forces are computed as normal force, tangential force and pitching moment. Indicial response means that the model works with a series of small disturbances.

The attached flow part reproduces the linear theory of [9] for force response for a pitching airfoil, a plunging airfoil and the wind gust case. See e.g. [8], section 2.6 for a general description of the attached flow solution.

The unsteady 2D airfoil aerodynamic response scales with time made dimensionless with speed relative to the airfoil and the chord of the airfoil. The time in the equations\(^1\) is therefore replaced with dimensionless time:

\(^1\) One exception is the “impulsive loads” which scale with the chord and the speed of sound
\[ s = \frac{V_{\text{rel}} \cdot t}{c/2} \] (2.1)

The model uses steady \( C_L(\alpha) \) and \( C_D(\alpha) \) data as input for a specific airfoil for which the dynamic forces are to be calculated. In the Beddoes-Leishman model, the forces in the body fixed system, \( C_N \) and \( C_T \), are used, where:

\[ C_N = C_L \cos(\alpha) + C_D \sin(\alpha) \] (2.2)

\[ C_T = C_D \cos(\alpha) - C_L \sin(\alpha) \] (2.3)

The Beddoes-Leishman model also treats the unsteady effects on the pitching moment. Dynamic stall has a dramatic effect on the pitching moment and on the damping for pitch-oscillations. This is very important for helicopter rotors but is less important for wind turbine blades. The treatment of the pitching moment is therefore left out in the description below.

The description below of the Beddoes-Leishman model follows as far as how to solve equations, the methods described in [5]. Alternative solutions using a state-space model are described in [7].

### 2.1 Attached flow

For attached flow, two different aspects are modeled. When the bound vorticity of the airfoil is time varying, vorticity is shed in the wake. This shed vorticity induces a flow over the airfoil, so that the airflow sensed by the airfoil is not the free stream velocity. If for example the lift (and the bound circulation of the airfoil) has been increasing for some time, then the shed wake will cause a downwash over the airfoil, resulting in less lift than would be anticipated in steady flow.
The other effect is the “impulsive load” effect. This effect is also called the apparent mass effect. This effect causes a lift force for a fast pitch or fast downward plunge motion.

The shed wake effects influences the “circulatory lift”. The “impulsive loads” are added to get the total attached flow lift (lift is here the same as normal force since the angle of attack is assumed small for the attached flow part)

The flow conditions are simulated by the superposition of indiciel responses.

**Circulatory part**

For the “circulatory” part this is written as:

\[
\Delta C_{N,c} = [C_{N\alpha}(M) \cdot \phi_c(s,M)] \Delta \alpha
\]  

(2.1.1)

The indicial response for the shed wake effects is approximated by

\[
\phi_c = 1 - A_1 \exp(-b_1 \beta s) - A_2 \exp(-b_2 \beta s)
\]  

(2.1.2)

where

\[
\beta = \sqrt{1 - M^2}
\]

Suggestions for constants \(A_1, A_2, b_1, \text{and} \ b_2\) are given in [5] as

\[
A_1 = 0.3 \quad A_2 = 0.7 \quad b_1 = 0.14 \quad \text{and} \quad b_2 = 0.53
\]

The shed wake effect is computed by using a lagged “effective” angle of attack: \(\alpha_e = \alpha - \alpha_i\) where \(\alpha_i\) can be seen as the shed wake induced angle of attack.

The numerical method to compute \(\alpha_i\) given in [5] is for the \(n\):th time step:

\[
X_n = X_{n-1} \exp(-b_1 \beta \Delta s) + A_1 \Delta \alpha_e \exp(-b_1 \beta \Delta s / 2)
\]  

(2.1.3)
\[
Y_n = Y_{n-1} \exp(-b_2 \beta \Delta s) + A_2 \Delta \alpha_n \exp(-b_2 \beta \Delta s / 2) \quad (2.1.4)
\]

\[
\alpha_{i,n} = X_n + Y_n \quad (2.1.5)
\]

The circulatory lift (or really the normal force) is taken as

\[
C_{N,c,n} = C_{Na} \left( \alpha_{E_n} - \alpha_0 \right) \quad (2.1.6)
\]

where \( \alpha_{E_n} = \alpha_n - \alpha_{i_n} \).

**Impulsive load**

The impulsive lift is calculated from piston theory:

\[
\Delta C_{N,l} = \left[ \frac{4}{M} \phi_l(s,M) \right] \Delta \alpha \quad (2.1.7)
\]

The transfer function is approximated by

\[
\phi_l(t) = e^{\frac{-t}{T_l}} \quad (2.1.8)
\]

where

\[
T_l = \frac{c}{a} \quad \text{where } c \text{ is the chord and } a \text{ is the speed of sound.}
\]

\[
K_a \quad \text{is a function of the mach number as given in [5].}
\]

Equation (2.1.7) and (2.1.8) are solved to give the impulsive lift at time step \( n \) as

\[
C_{N,l,n} = \frac{4 \cdot K_a \cdot c}{V_{rel}} \left( \frac{\Delta \alpha_n}{\Delta t} - D_n \right) \quad (2.1.9)
\]

where the deficiency function is given by
\[
D_n = D_{n-1} \cdot e^{\frac{-\Delta t}{K_n \cdot \tau}} + \left( \frac{\Delta \alpha_{n} - \Delta \alpha_{n-1}}{\Delta t} \right) \cdot e^{\frac{-\Delta t}{2K_n \cdot \tau}} \tag{2.1.10}
\]

**Total attached flow lift**

The total attached flow lift (potential flow lift) is then obtained by adding the circulatory and impulsive lift.

\[
C_{N,\text{pot}} = C_{N,c} + C_{N,l} \tag{2.1.11}
\]

### 2.2 Separated flow

The simulated separated flow effects are: 1) a dynamic delay in the movement of the boundary layer separation position and 2) the dynamic leading edge stall with a vortex travelling downstream along the chord.

The first part is based on the physical concept that \( C_N \) is a function of the separation point position and that the movement of the separation point has a dynamic delay.

The second part results in “vortex lift” that is added to the normal force. The “vortex lift” has a dramatic effect on the pitching moment. Much attention has therefore been given to estimate the conditions at which the vortex lift is shed over the airfoil and the speed at which it travels. The condition at which the dynamic stall vortex is shed, is in the model controlled by the condition that the normal force coefficient exceeds a certain value. The condition at when leading edge stall occurs, is really controlled by a critical leading edge pressure coefficient. To link this critical leading edge pressure coefficient to \( C_N \), the model works with a lagged \( C_N \). (See e.g. [5] for details and [10] for comments on the difference between plunge and pitching motion).
Delay in the movement of the separation point.

To model the effect of the delay in the movement of the separation point position requires a $C_N(f)$ relationship. One such relationship is the Kirchoff flow model as described in [5].

The relationship between $f$ and $C_N$ in the Kirchoff flow model is:

$$C_N(\alpha) = C_{N\alpha} \cdot 0.25 \cdot \left( 1 + \sqrt{f(\alpha)} \right)^2 (\alpha - \alpha_0)$$  \hspace{1cm} (2.2.1)

Where $\alpha_0$ is the angle of attack for zero lift and $C_{N\alpha}$ is the attached flow lift curve slope. $f$ is the chord-wise position of separation. $f$ is made dimensionless with the chord length and $f = 1$ represents fully attached flow and $f = 0$ represents fully separated flow.

The static $f(\alpha)$ can in principle be obtained as a table by solving equation (2.2.1) with the steady $C_N(\alpha)$ curve. However, in the Beddoes-Leishman model as described in [5], a parametric $f(\alpha)$-curve that is a best fit to the steady $C_N(\alpha)$ curve is suggested.

In order to obtain the dynamic value of the position of separation point, the model is that the dynamic $f$ lags behind its steady value according to a first order lag equation:

$$\frac{df}{ds} = \frac{f_{\text{stat}} - f}{T_f}$$  \hspace{1cm} (2.2.2)

$s$ is here the dimensionless time from equation (2.1). $T_f$ is a semi-empirical time constant (time constant in the $s$-space).

By using the dimensionless time, $s$, the model for the lag in the separation point works for any frequency of oscillation or any time history with $T_f$ as a constant.

The dynamic lift, $C_{N,f}$, is obtained from the Kirchoff flow equation with the dynamic value of the separation point position.
Vortex lift

The model further includes a model for vortex lift, with the amount of vortex lift determined by a time-lag function with a time constant, $T_v$. It is assumed that the vortex lift contribution can be viewed as an excess circulation, which is not shed into the wake until some critical condition is reached. The vortex lift, $C_{N,v}$, is determined by the following equations:

The “feed” of vortex lift is proportional to the difference in unsteady circulatory lift and the non-linear lift given from the Kirchoff flow equation with the dynamic value of the separation point position.

$$c_v = C_{N,v} - C_{N,f}$$  \hspace{1cm} (2.2.3)

At the same time, the total accumulated vortex lift is allowed to decay exponentially with time, but may also be updated with new vortex lift feed.

$$\frac{dC_{N,v}}{ds} = \frac{dC_{\infty}}{dt} - \frac{C_{N,v}}{T_v}$$  \hspace{1cm} (2.2.4)

The last equation is solved at the $n$:th time step as:

$$C_{N,v,n} = C_{N,v,n-1} e^{-\frac{\Delta t}{T_v}} + (c_{v,n} - c_{v,n-1}) e^{-\frac{\Delta t}{T_v}}$$  \hspace{1cm} (2.2.5)

Conditions for when the addition of new vortex lift feed should stop and conditions for how $T_v$ should vary with $f$ and the position of the travelling vortex, are given in [5].

2.3 The model in steps

The model works as an open loop model so that the whole model is programmed in steps in which the output from earlier sub-models are input to the next sub-model.
Step 1
Calculate attached flow effects (impulsive loads and shed wake effects) => effective angle of attack, $\alpha_E$ and impulsive force $C_{N,imp}$

Step 2
Compute a shift in angle of attack due to the lag in leading edge pressure response.

$$\frac{dC_{N, pot}'}{ds} = \frac{C_{N, pot} - C_{N, pot}'}{T_p}$$  \hspace{1cm} (2.3.1)

Where $T_p$ is an empirical time constant.

$C_{N, pot}'$ is then used to get substitute value of the effective angle of attack:

$$\alpha_f = \frac{C_{N, pot}'}{C_{Na}} - \alpha_0$$  \hspace{1cm} (2.3.2)

Step 3
Use $f_{stat} = f_{stat}(\alpha_f)$ and compute the unsteady separation point position, $f_{dyn}$, from the lag equation for the separation point. Use $f_{dyn}$ in the Kirchoff $C_n(f)$ relationship to obtain a dynamic value of the normal force coefficient, $C_{N,f}$.

Step 4
Compute vortex lift from equations (2.2.3) and (2.2.4)

Step 5
Add components to get total normal force:

$$C_N = C_{N,f} + C_{N,v} + C_{N,v}$$
2.4 Semi-empirical constants

The main semi-empirical constants are the three time constants for the leading edge pressure lag, $T_p$, the lag in separation point movement, $T_f$, and for the vortex lift decay, $T_v$.

Values for these constants are suggested in [5].

In the Beddoes-Leishman model, semi-empirical constants are also needed to control the conditions for the dynamic stall vortex shedding and to control its travel speed. For an explanation of these constants, $C_{N,i}$ and $T_{id}$, see [5].

In [5], several conditions are also suggested for which some of the time constants should be halved or doubled depending on e.g. the value of the separation point position, $f$ or the position of the traveling vortex.
3 The DYNSTALL unsteady airfoil aerodynamics model

The model in the DYNSTALL-package is an implementation of the Beddoes-Leishman model, but with a few changes made relative to the description above and as given in [5], [6] and [7].

The changes to the model are made to make it more robust for use in wind turbine aeroelastic simulations with turbulent wind. The model now works for angles of attack in the whole range [-180°,180°]. Some changes were also introduced during the “Stallvib” project [3] where the unsteady aerodynamics for lead-lag blade oscillations where studied.

In the following text, when there is mention of the current version of DYNSTALL it is referred to the subroutines in the files:

dyncl_c.f, v_esti_c.f and vo_sub_c.f

3.1 A change to work in the wind reference system

The Beddoes-Leishman model as described in [5] works in the body fixed frame. Forces are described in the direction normal to the airfoil, $C_N$, and in the chord-wise direction, $C_T$. The attached flow part originates from linearised theory at small angles of attack where $\alpha \approx \sin(\alpha) \approx \tan(\alpha)$, so that no distinction can really be made between $C_N$ and $C_L$ in that theory. For the non-linear part (separated flow) the Beddoes-Leishman model also works with $C_N$ and $C_T$.

There might be advantages by setting up the separated flow equations in the body fixed system but difficulties arise when the dynamic behavior of $C_T$ at high angles of attack should be modeled with the model from [5].

The current version of DYNSTALL therefore uses the wind reference system. Most equations in $C_N$ in the Beddoes-Leishman
model are treated as being equations in $C_L$. One exception is that the vortex lift still is treated as being normal to the airfoil chord.

One advantage of working in the wind reference system is that the delay in the separation point position automatically affects the tangential force. This happens since $C_T$ has a rather large component of $C_L$ at stall angles of attack. Most of the effect of the dynamic delay of the separation point on $C_T$ is therefore automatically a result of the delay in the lift. The effect of the dynamic delay of the separation point on $C_D$ is also modeled in DYNSTALL. Still the main effect on $C_T$ comes from the lift part at stall angles of attack.

The vortex lift is, however, assumed to act only in the normal force direction. An increase of $T_f$ will increase the dynamic stall loop width of the $C_N(\alpha)$-curve as well as the $C_T(\alpha)$-curve, whereas an increase of $T_V$ will increase only the width of the $C_N(\alpha)$-curve and leave the $C_T(\alpha)$-curve unaffected. The fact that $T_V$ only affects $C_N$ makes it possible to tune the time constants, $T_f$ and $T_V$, to obtain a good fit to both the $C_T(\alpha)$ as well as the $C_N(\alpha)$-curve.

### 3.2 Attached flow

The attached flow equations are basically the same as in [5] and given in chapter 2 above.

In DYNSTALL, two methods are however available for how to calculate the shed wake effect. In the alternative method explained in section 3.2.2 the shed wake effect is made a function of the actual circulation history and not the circulation history that would occur for attached flow as in the original model.

#### 3.2.1 The angle of attack

The angle of attack that is input to the equations includes the pitch rate effects.
A geometrical angle of attack, $\alpha_g$, is defined as the angle between the chord-line and the relative velocity to the airfoil. The relative velocity to the airfoil includes here the translation motion of the airfoil. No difference is hence made between the airfoil moving in still air or the airfoil being at a stand-still at an incidence in an airflow.

In the picture above, the airfoil is at a geometrical incidence and also has a heaving velocity, $u_y$, and a lead lag velocity, $u_x$. $\alpha_g$ is then calculated from the angle of the relative velocity to the chord line as:

$$\tan(\alpha_g) = \frac{V_y - u_y}{V_x - u_x} \quad (3.2.1.1)$$

In order to automatically include the pitch rate effect in the lift, the angle of attack at the $\frac{3}{4}$ chord position can be used. The DYNSTALL package therefore also works with the angle of attack at the $\frac{3}{4}$ chord position. This angle of attack is called $\alpha_{75}$.

With the velocity relative to the airfoil, $V_{rel}$, and the pitch rate $q$, then

$$\alpha_{75} = \alpha_g + \frac{c \cdot q}{2 \cdot V_{rel}} \quad (3.2.1.2)$$
\( \alpha_{75} \) is then used for \( \alpha \) in equations for the circulatory lift in section 2.1.

### 3.2.2 Attached flow equations (LPOTMETH=3)

The attached flow equations are available in two versions. The choice of version is controlled by an input parameter \( \text{LPOTMETH} \). (\( \text{LPOTMETH}=1 \) used to be the attached flow equations in section 2.1. but is no longer implemented since it is equivalent to \( \text{LPOTMETH}=3 \) for constant \( V_{rel} \)).

**Circulatory part**

The equations are reformulated to work with the normal velocity at the \( \frac{3}{4} \) point chord position. This is the same as reformulating the equations in circulation rather than in lift coefficient. With this formulation, the circulatory effect will be correct for lead-lag motions and for a varying wind speed.

Equation (2.1.1) then becomes

\[
\Delta \Gamma = \left[ \frac{c}{2} \cdot C_{La} \cdot \phi_c \right] \Delta w \quad (3.2.2.1)
\]

\( w \) is the normal component of the relative velocity to the airfoil at the \( \frac{3}{4} \) point chord position. In this case the normal direction is taken as normal to the zero-lift line and the small angle approximation is used.

\[
w = \frac{(\alpha_{75} - \alpha_n)}{V_{rel}} \quad (3.2.2.2)
\]

The equations to solve for the effective angle of attack are then at the \( n \):th time step

\[
X_n = X_{n-1} \exp(-b_1 \beta \Delta s) + A_1 \Delta w_n \exp(-b_1 \beta \Delta s / 2) \quad (3.2.2.3)
\]

\[
Y_n = Y_{n-1} \exp(-b_2 \beta \Delta s) + A_2 \Delta w_n \exp(-b_2 \beta \Delta s / 2) \quad (3.2.2.4)
\]
\[ \alpha_E = (\alpha_{75} - X_n - Y_n) / V_{rel} + \alpha_0 \]  \hspace{1cm} (3.2.2.5)

This method is obtained by choosing LPOTMETH=3.

\[ \beta = \sqrt{1 - M^2} \]  \hspace{1cm} is in the current version set to unity since Mach number effects for wind turbines for the shed wake effect are assumed small.

In order to avoid queer results when the linearised theory is used at high angles of attack, \( \Delta w \) in equations (3.2.2.3) and (3.2.2.4) are factored with a factor \( fade \). Where

\[ fade = \cos^2(\alpha_{75}) \]  \hspace{1cm} (3.2.2.6)

**Impulsive load**

The impulsive load should be calculated from the airfoil “plunge” velocity at the \( \frac{3}{4} \) point chord position. This plunge velocity could come from airfoil plunging but could also be due to a pitch rate induced velocity at the \( \frac{3}{4} \) point chord position.

With, \( u_p \) as the velocity of the \( \frac{3}{4} \) point chord position directed in the negative y-direction, the equations for impulsive loads at the \( n \):th time step are written:

\[ C_{L,n} = \frac{4 \cdot K_a \cdot \xi}{V_{rel,n} \cdot U_{mean}} \left( \frac{\Delta u_{p,n}}{\Delta t} - D_n \right) \]  \hspace{1cm} (3.2.2.7)

with the deficiency function given by

\[ D_n = \frac{\Delta u_{p,n} - \Delta u_{p,n-1}}{\Delta t} \cdot e^{-\Delta t / \frac{\tau_k \cdot f}{\Delta t}} \]  \hspace{1cm} (3.2.2.8)

and \( U_{mean} \) is the mean of \( V_{rel} \) during the current and the previous time step.
$K_\alpha$ is in the current version of DYNSTALL set to 0.846 which is the value taken from [5] for $M=0.15$.

In the current version the impulsive normal force from equation (3.2.2.7) is taken as the impulsive lift force even though this is strictly correct only for $\alpha = 0$. In order to avoid queer results when the linearised theory is used at high angles of attack, $\Delta u_p$ in equations (3.2.2.7) and (3.2.2.8) are factored with the factor $fade$ from equation (3.2.2.6).

**Pitching moment**

The attached flow effects on the pitching moment are included by adding the impulsive load component to the pitching moment

$$C_{m,l} = \frac{C_{L,l}}{4} \quad (3.2.2.9)$$

**3.2.3 Shed wake effects with account for a varying lift curve slope (LPOTMETH=4)**

The above equations (used with LPOTMETH=3) are valid for attached flow. The shed wake effect is then calculated as if the flow were attached with a lift curve slope $C_{L,\alpha}$, which is the lift curve slope around the zero lift angle of attack.

At angles of attack where the airflow starts to separate, the steady lift curve slope is no longer the same as for attached angles of attack. The lift may even decrease for an increase in angle of attack above stall.

It is then questionable if the shed wake effect (circulatory lift) could be calculated with a constant $C_{L,\alpha}$.

On way to overcome this would be to calculate the shed wake effects from the circulation history (or lift history for constant $V_{rel}$) instead.
One problem is, however, that the lift is not known until the dynamic effects on the separation point positions is determined. This is solved by first calculating an estimate of $C_{L,f}$. This estimate, $C_{L,f,est}$, is calculated as in step 2 and 3 of the Beddoes-Leishman model but as explained in the sections on separated flow below.

The shed wake effect is then made a function of the difference in $C_{L,f,est}$ instead of the difference in $\alpha_{75}$:

$$\Delta C_{L,f,est,n} = C_{L,f,est,n} - C_{L,f,est,n-1}$$

Further, $C_{L,f,est}$ must be based on the angle of attack without the shed wake effects if the model should give correct results for attached flow with the equations corresponding to (3.2.2.3)-(3.2.2.5). $\alpha_E$ is therefore set to $\alpha_{75}$ when the estimate of the dynamic lift, $C_{L,f,est}$, is calculated.

The method used to account for the fact that a reduced lift curve slope due to separated flow give reduced shed wake effects is: exchange $\Delta \alpha$ in equations (2.1.1)-(2.1.4) with $\Delta C_{L,f,est}/C_{La}$.

To, at the same time account for a varying circulation due to a varying velocity, $\Delta w$ is used instead of $\Delta \alpha$ with $w$ as in equation (3.2.1.2)

$$\Delta w$$ to be put in equations (3.2.1.3)-(3.2.2.5) is then calculated, by differentiation, as

$$\Delta w = \frac{C_{L,f,est,mean}}{C_{La}} \cdot \Delta V_{rel} + \frac{\Delta C_{L,f,est}}{C_{La}} \cdot U_{mean}$$

with $C_{L,f,est,mean}$ being the mean of $C_{L,f,est}$ at the current and the previous time step.

As for LPOTMETH=3 $\Delta w$ is multiplied with the factor fade from equation (3.2.2.6)
The so calculated effective angle of attack will be exactly the same as calculated with LPOTMETH=3 if the flow is attached. Attached flow means here that $f=1$ and $C_{L,f}(\alpha_f) = C_{L,att}(\alpha_f)$.

For separated flow it will approximately account for the fact that the shed wake history is a function of the lift history rather than of the angle of attack history. The lift that is used to estimate the change on lift excludes the vortex lift since the vortex lift is considered not to affect the shed wake history.

The extra computing for LPOTMETH=4 relative to LPOTMETH=3, is that step 2 and 3 has to be run an extra time every time step to obtain $C_{L,f,est}$.

3.3 Separated flow

3.3.1 A shift in the angle of attack

The separation point position is statically a function of the angle of attack. What really influences the separation position is the pressure gradients in the boundary layer. To use the shifted angle of attack, $\alpha_f$, as calculated from equation (2.3.2) is a way to account for the unsteady pressure gradient.

In DYNSTALL, the shift in the angle of attack to compensate for the lag in the leading edge pressure is used just as in the original model.

What is new to the FFA version, is to account for changes in the unsteady pressure gradient due to a varying free stream velocity. This was introduced during the STALLVIB project where lead-lag airfoil oscillations were studied [3], [11].

By studying the unsteady Bernoulli equation it can be seen that the pressure gradient along the airfoil will be different if the relative velocity is constant, increasing or decreasing (see [3] or [11]). If $V_{rel}$
is increasing, then the “effective” unsteady pressure gradient is found to be more favorable which in principle should result in less separation.

The question is how this effect on the pressure gradient and the corresponding effect on the separation point should be incorporated in the dynamic stall model. One way to do this is to see the resulting pressure gradients as gradients occurring at a different angle of attack. (A lower angle of attack for an accelerating free stream.) The point of separation, \( f \), could then be determined using a corrected angle of attack as step 2b in the dynamic stall model. The assumption is that, at some angle of attack, \( \alpha = \alpha + \Delta \alpha \), the pressure gradient for the case with varying \( V_{rel} \), is similar to the pressure gradient at an angle of attack \( \alpha \), for the case with constant \( V_{rel} \).

The equations to derive an estimate of the shift are given in [11] and [3]. The result is that the shift should be proportional to the dimensionless velocity change rate

\[
\gamma = \frac{c}{2 \cdot V_{rel}^2} \cdot \frac{\partial V_{rel}}{\partial t} \quad (3.3.1.1)
\]

The amount of shift is further controlled by an empirical parameter, \( f_u \).

An estimate of \( f_u = 0.5 \) is given in [11].

Step 2, as in the description of the Beddoes-Leishnan model in section 2.3, is in the FFA model:

Equation (2.3.1) is solved at the \( n \):th time step as

\[
dp_n = dp_{n-1} \cdot e^{\frac{-\Delta t}{T_p}} + \left( C_{N,\text{par},n} - C_{N,\text{par},n-1} \right) e^{\frac{-\Delta t}{T_p}} \quad (3.3.1.2)
\]

and
\[ C_{N,\text{pot,n}} = C_{N,\text{pot,n}} - \text{fade} \cdot dp_n \] (3.3.1.3)

The factor \textit{fade} from equation (3.2.2.6) is included since the concept anyway breaks down at high angles of attack.

The shifted angle of attack \( \alpha_f \) is taken from equation (2.3.2)

In the FFA-model the angle of attack is also shifted due to a varying velocity. \( \alpha_f \) is therefore taken as

\[ \alpha_f = \frac{C_{N,\text{pot}}}{C_{Na}} - \alpha_0 - \gamma \cdot f_u \] (3.3.1.4)

### 3.3.2 Lift as function of the separation point position

The steady \( C_l(\alpha) \) curve is in the dynamic stall model exchanged with a static \( f(\alpha) \) relationship as explained in section 2.2.

The dynamic value (unsteady value) of the lift coefficient is then obtained by putting in a dynamic value of \( f \) in the \( f(\alpha) \) relationship.

Two different types of \( f(\alpha) \) can be used in DYNSTALL.

The choice between models is controlled by a variable \texttt{lfmeth}.

One model is the relationship introduced by Øye [12]. The other is a version of the Kirchoff flow model.

In either case, the dynamic value of \( f \) is calculated by solving equation (2.2.2)

#### 3.3.2.1 The Øye separation point model (lfmeth=2)

The formula linking \( C_L, f \) and \( \alpha \) originating from Øye is used.

\[ C_L = f \cdot C_{L,\text{inv}} + (1-f) \cdot C_{L,\text{sep}} \] (3.3.2.1.1)
This relationship requires steady curves of $C_{L,\text{inv}}(\alpha)$ and $C_{L,\text{sep}}(\alpha)$ in order to obtain either a steady $C_{L,\text{stat}}(\alpha)$ curve from a steady $f(\alpha)$ or a dynamic $C_L$ from a dynamic $f$.

Equation (3.3.2.1.1) can be used to find a static $f(\alpha)$ relationship.

$$f_{\text{stat}}(\alpha) = \frac{C_{L,\text{stat}}(\alpha) - C_{L,\text{sep}}(\alpha)}{C_{L,\text{inv}}(\alpha) - C_{L,\text{sep}}(\alpha)}$$  \hspace{2cm} (3.3.2.1.2)

$C_{L,\text{inv}}(\alpha), C_{L,\text{sep}}(\alpha)$ and $f_{\text{stat}}(\alpha)$ are input to the program as tabulated values as function of $\alpha$.

$f$ should perhaps not be seen strictly as the separation point position, but rather as an interpolation factor between $C_{L,\text{inv}}(\alpha)$ and $C_{L,\text{sep}}(\alpha)$.

$C_{L,\text{inv}}$ could be taken as $C_L(\alpha - \alpha_0)$ and $C_{L,\text{sep}}$ as the curve for fully separated flow. The choice for the latter curve is more arbitrary. Øye [12] suggests a curve a curve starting with a slope of half value of the unseparated curve and gradually fitting to the steady $C_L(\alpha)$ curve at approximately 30° angle of attack.

To solve the differential equation (2.2.2), a value of $f'$ is obtained from the $f_{\text{stat}}(\alpha)$ table with the angle of attack $\alpha_f$ as input.

(2.2.2) is then solved at the $n$:th time step as

$$df_n = df_{n-1} \cdot e^{\frac{-\Delta t}{T_f}} + (f_n' - f_{n-1}') \cdot e^{\frac{-\Delta t}{T_f}}$$ \hspace{2cm} (3.3.2.1.3)

$$f_{\text{dyn,n}} = f_n' - df_n$$ \hspace{2cm} (3.3.2.1.4)

$C_{L,f}$ is obtained from equation (3.3.2.1.1) as

$$C_{L,f}(\alpha_g) = f_{\text{dyn}} \cdot C_{L,\text{inv}}(\alpha_g) + (1 - f_{\text{dyn}}) \cdot C_{L,\text{sep}}(\alpha_E)$$ \hspace{2cm} (3.3.2.1.5)
### 3.3.2.2 A version of the Kirchoff flow (Ifmeth=4)

The Kirchoff flow equation written in equation (2.2.1) is used, but with some modifications.

First, $C_L$ is used rather than $C_N$.

With $f=1$ the unmodified Kirchoff flow equation with $C_L$ will give rather high values of $C_L$ at large angles of attack. The DYNSTALL model should work over the whole range of angles of attack, and modifications are introduced to limit queer results at high angles of attack. The Kirchoff flow equation is therefore modified to limit $C_L$ for $f=1$. Another fix is introduced to overcome the fact that if equation (2.2.1) (with $C_N$ replaced by $C_{L,\text{stat}}$) is used to solve for the static $f_{\text{stat}}(\alpha)$, then this will only work as long as

$$C_{L,\text{stat}} < \frac{1}{4} \cdot C_{La} (\alpha - \alpha_0).$$

With $C_{L,\text{inv}} = C_{La} \cdot (\alpha - \alpha_0)$ and $C_{L,\text{sep}} = C_{L,\text{inv}} / 4$, the Kirchoff equation can be written to use the expressions $C_{L,\text{inv}}(\alpha)$ and $C_{L,\text{sep}}(\alpha)$ as in the Øye model.

$$C_L = C_{L,\text{sep}} + C_{L,\text{inv}} \cdot \frac{1}{4} \cdot (f + 2 \sqrt{f}) \quad (3.3.2.2.1)$$

To limit $C_L$ for $f=1$, equation (3.3.2.2.1) is modified for $\alpha > \alpha_c$.

If $\alpha > \alpha_c$, then the linear $C_{L,\text{inv}} = C_{La} \cdot (\alpha - \alpha_0)$ is exchanged for a sine curve that bends down almost to zero at $90^\circ$:

$$C_{L,\text{inv}} = \frac{1}{d_1} \cdot \sin(d_1 \cdot (\alpha - \alpha_0)) + d_2 \quad (3.3.2.2.2)$$

$d_1$ is a constant, somewhat arbitrarily, chosen to be 1.8. The constant $d_2$ assures continuity of the two $C_{L,\text{inv}}$ expressions at $\alpha = \alpha_c$ with
The original Kirchoff curve is thus retained unchanged for $\alpha < \alpha_c$. $\alpha_c = 30^\circ$ is used in the AERFORCE package [1].

In order to get the equation to work for angles of attack where $\frac{1}{4} \cdot C_L (\alpha - \alpha_0) > C_{L,stat}$, $C_{L,sep}$ is then set to $C_{L,stat}$ and $f_{stat}$ is then set to zero.

To construct tables of $C_{L,inv}(\alpha)$, $C_{L,sep}(\alpha)$ and $f_{stat}(\alpha)$ for the modified Kirchoff flow model:

If $\alpha < \alpha_c$,

$$C_{L,inv} = C_L \alpha \cdot (\alpha - \alpha_0)$$

Else $C_{L,inv}$ is taken from equation (3.3.2.2).

Equation (3.3.2.2.1) is used to solve for $f_{stat}$ if $C_{L,stat} > \frac{1}{4} \cdot C_L (\alpha - \alpha_0)$.

Then $C_{L,sep} = C_{L,inv} / 4 \Rightarrow$

$$f_{stat} = \left( \sqrt{\frac{4 \cdot C_{L,stat}}{C_{L,inv}}} - 1 \right)^2$$

(3.3.2.2.4)

If $C_{L,stat} < \frac{1}{4} \cdot C_L (\alpha - \alpha_0)$, then

$$f_{stat} = 0 \text{ and } C_{L,sep} = C_{L,stat}.$$
\[ C_{L,f}(\alpha_c) = C_{L,sep}(\alpha_e) + \frac{1}{4} \cdot C_{L,inv}(\alpha_e) \cdot (f_{dyn} + 2 \sqrt{f_{dyn}}) \]  
(3.3.2.5)

with \( f_{dyn} \) from equations (3.3.1.3) and (3.3.1.4)

### 3.3.3 Vortex lift (lvormeth=2)

The vortex lift is calculated with equations equivalent to equations (2.2.3)-(2.2.5) but with equation (2.2.3) replaced with

\[ c_v = C_{L,c} - C_{L,f} \]  
(3.3.3.1)

The feed in vortex lift is further only allowed as long as the angle of attack is increasing. So that the second term in equation (2.2.5) is zero if the angle of attack is decreasing.

Furthermore, negative feed of vortex lift is not allowed. So that if \((c_{v,r} - c_{v,r-1}) < 0\), the second term in equation (2.2.5) is again set to zero.

The feed in vortex lift is furthermore disregarded if the angle of attack is too high. This is controlled by an input variable \( \alpha_{\text{shift,ds}} \).

In AERFORCE [1] \( \alpha_{\text{shift,ds}} \) is set to 50° (0.87 rad).

### 3.3.4 An alternative version of vortex lift (lvormeth=1)

The subroutine vo_sub_c.f also includes a version of vortex lift calculations where the feed in vortex lift stops after a certain time delay after a critical \( C_N \) condition has been obtained.

This version of vortex lift calculations also includes criteria for when vortex feed can start again after having been terminated.

Only primary vortex shedding is considered. The method is considered less robust for simulations with stochastic wind for wind turbines. No further explanation of the method is therefore given in this report, and the reader is referred to the code and to [5].
3.4 Unsteady drag

Unsteady drag effects are implemented in DYNSTALL. The unsteady effects on the drag are:

- $C_{D,\text{ind}}$ · The shed wake induced drag
- $C_{D,\text{sep}}$ · The change in drag due to the separation point position being different from its static position.
- $C_{D,\text{vor}}$ · “Vortex drag”. A component of the vortex lift must be added to the drag since vortex lift is assumed to act normal to the chord.

The total drag is taken as:

$$ C_D = C_{D,\text{stat}} + C_{D,\text{ind}} + C_{D,\text{sep}} + C_{D,\text{vor}} $$  \hfill (3.4.1)

**Induced drag**

$C_{D,\text{ind}}$ is calculated as the component of lift inclined the shed wake induced angle of attack

$$ C_{D,\text{ind}}(\alpha_g) = \sin(\alpha_{75} - \alpha_E) \cdot C_{L,f}(\alpha_g) $$  \hfill (3.4.2)

**Separation drag**

If the separation point is forward of its static position, then it is assumed that the drag will be less than if the separation point was at its static position and vice versa. This is modeled, more or less as suggested by Montgomerie in [13]:

$$ C_{D,\text{sep}}(\alpha_g) = A_{cd} \cdot (C_{L,\text{stat}}(\alpha_E) - C_{L,f}(\alpha_g)) $$  \hfill (3.4.3)

$A_{cd}$ is an empiric constant.

**Vortex drag**

The vortex drag is calculated as
With the vortex lift taken as

\[ C_{L_{\text{vor}}} (\alpha_g) = C_{N_{\alpha}} (\alpha_g) \cdot \cos(\alpha_g) \]  

(3.4.5)

the tangential force component of vortex lift will then be zero.

**Versions of how to add dynamic drag effects**

An input parameter controls how the unsteady drag effects should be calculated.

If \( LCDDYN=0 \), then the drag is taken as the static drag

\[ C_D (\alpha_g) = C_{D_{\text{stat}}} (\alpha_g) \]

If the unsteady drag is modeled then it is not quite clear if the static drag from the tables of \( C_{D_{\text{stat}}} (\alpha) \) should be taken for the geometric angle of attack or for the effective angle of attack, \( \alpha_E \). There are therefore two versions:

**LCDDYN=1 :**

The static drag is evaluated at the geometric angle of attack:

\[ C_D (\alpha_g) = C_{D_{\text{stat}}} (\alpha_g) + C_{D_{\text{ind}}} + C_{D_{\text{sep}}} + C_{D_{\text{vor}}} \]  

(3.4.6)

**LCDDYN=2 :**

The static drag is evaluated at the effective angle of attack:

\[ C_D (\alpha_g) = C_{D_{\text{stat}}} (\alpha_E) + C_{D_{\text{ind}}} + C_{D_{\text{sep}}} + C_{D_{\text{vor}}} \]  

(3.4.7)
3.5 Unsteady pitching moment

Unsteady effects on the pitching moment for separated flow effects are not included in DYNSTALL.

The pitching moment that is returned is thus the static pitching moment corrected for attached flow unsteady effects.

\[ C_M(\alpha) = C_{M,\text{stat}}(\alpha) + C_{M,u} \quad (3.5.1) \]

with \( C_{M,u} \) from equation (3.2.9)

3.6 Ways to make the model work for all angles of attack

For simulation of wind turbine loads in design calculations, the wind can in principle come from any direction. The airfoils could therefore be blown at from the rear.

One problem in itself is to generate sensible airfoil data for the whole range of angles of attack. The other problem is to make the dynamic stall model to work for flow (that is 2D flow) at any angle.

First all equations in sections 3.1-3.4 are written so that the dynamic stall works for negative angles of attack in the range \([-90^\circ, 0^\circ]\). This comes automatically and only needs some special treatment in the vortex lift calculations. With a decrease in angle of attack in section 3.3.3 is really meant a decrease of absolute values of angle of attack and negative feed of vortex lift refers to the case when the angle of attack is positive.

Moreover, the equations should work for the angle of attack ranges \([-180^\circ, -90^\circ]\) and \([90^\circ, 180^\circ]\).

The angle of attack range \([-90^\circ, 90^\circ]\) is called \(\alpha_{\text{case}} = 1\) in the code.
The angle of attack range $[-180^\circ,-90^\circ]$ is called $\alpha_{\text{case}} = 2$ in the code. The angle of attack range $[90^\circ,180^\circ]$ is called $\alpha_{\text{case}} = 3$ in the code.

The angle of attack ranges for $\alpha_{\text{case}} = 2$ and $\alpha_{\text{case}} = 3$ could also be put together in a continuous angle of attack range with the transform

\[
\alpha_2 = \alpha + 180^\circ \text{ if } \alpha_{\text{case}} = 2
\]

\[
\alpha_2 = \alpha - 180^\circ \text{ if } \alpha_{\text{case}} = 3
\]

(3.6.1)

(3.6.2)

In this new angle of attack, both $\alpha = -180$ and $\alpha = +180$, is represented by $\alpha_2 = 0$.

A second lift curve slope and a second zero lift angle of attack, $C_{L_{\alpha,2}}$ and $\alpha_{0,2}$ must therefore be used if $\alpha$ is in the range $[-180^\circ,-90^\circ]$ or $[90^\circ,180^\circ]$.

For every angle of attack there, thus, also exist a “dynamic stall angle of attack” that goes in the equations for the dynamic stall model. Since the dynamic stall model includes some table looking, the, original angle of attack also is passed to the model.

The dynamic stall model is therefore fed with

$\alpha_g$, $\alpha_{g,ds}$ and $\alpha_{75,ds}$. $\alpha_{g,ds}$ and $\alpha_{75,ds}$ are the $\alpha_2$ angles of attack from equations (3.5.1) and (3.5.2) if $\alpha_{\text{case}} = 2$ or $\alpha_{\text{case}} = 2$. If $\alpha_{\text{case}} = 1$, then $\alpha_{g,ds} = \alpha_g$ and $\alpha_{75,ds} = \alpha_{75}$.

$C_{L_{\alpha}}$ and $\alpha_0$ are also chosen for the respective $\alpha_{\text{case}}$.

3.7 Robustness of the DYNSTALL-methods

Differential equations are solved such that rather long time steps could be used. To get the method work well, the time step should of course be shorter than the time-constants in the equations. The equations are, however, written such that if the time constant is small
in relation to the time step, then the dynamic effects just becomes absent. See e.g. equations (3.3.1.2) and (3.3.1.3) or the equations to calculate the shed wake effects, equations (3.2.2.3) and (3.2.2.5).

In order to simulate the attached flow effects well the time step should be approximately shorter than corresponding to $\Delta t = 0.5$ ($\Delta t = 0.5$ is the step length for the airfoil to move a $\frac{1}{4}$ chord).

### 3.7.1 Versions that work for all angles of attack

If the dynamic stall model should be used for angles of attack above the range $[-90^\circ, 90^\circ]$, the Øye model (lfmeth=2) does not work.

The modified Kirchoff model (lfmeth=4) works for the whole range of angles of attack.
4 Input to the FORTRAN subroutine package

The use of the subroutines for dynamic stall is partly described in the description of the AERFORCE blade-element/momentum code program [1]. Much of the same information and some further information is given below.

4.1 Subroutines

The main subroutine in the DYNSTALL-package is the `dyncl_c` subroutine.

This subroutine then in turn calls subroutine `v_esti_c` in file `v_esti_c.f` and the appropriate subroutine for vortex lift in the file `vo_sub_c.f`.

The subroutines work with a number of state-variables. E.g. $d_s$ and $C_{N, pot}$ at the current time step and at the previous time step (see equation (3.3.1.2). Methods to solve differential equations are such that data only from the previous and current time step are needed).

In order to update values at the current time step to values at the next time step subroutine `cu_2_old_aer_c` in the file `cu_2_old_aer_c.f` is called.

The DYNSTALL-package is intended for use in a blade-element/momentum code and it is called for one blade element at the time. Matrix indices for blade and radial element number are therefore needed (variables IB and IR).

At the first time step, values for state-variables at the previous time step must be set. An example of how this can be made is given in the subroutine `clcd_firstime` in the AERFORCE subroutine package.
4.2 Conventions for variable names.

For the solution of differential equations, variable values at the previous time step is needed. Variable names ending with \_1 refers to variable values at the previous time step.

4.3 About history of variable names

In the code, variable names still bear marks from the first versions of DYNSTALL which worked with \( C_N \) and instead of \( C_L \). A variable CNF should perhaps better be called CLF now, and CNSEPIN should perhaps better be called CLSEPIN. The history of the code is hence, not for best readability, still visible.

4.4 Communication with the subroutines

The basic input to the main DYNSTALL subroutine, dyncl_c, is

- Airfoil data
- Angle of attack, chord and time-step

The basic output is

- Lift, drag and pitching moment coefficients

Some of the variables are communicated as formal parameters to dyncl_c, but the main part are communicated via common areas.

The formal parameters in the subroutine call are explained in the code.

The dyncl_c subroutine is called from subroutine clcdcalc_c in the AERFORCE subroutine package and this gives some information on the use.
4.5 Common areas in Dyncl_c.inc

The file `dyncl_c.inc` includes common-areas with variables that are needed by the dynamic stall subroutines e.g. storage of airfoil aerodynamic coefficients.

4.5.1 Storage of static airfoil data.

The aerodynamic static airfoil data should be stored in data tables $C_L(\alpha)$, $C_D(\alpha)$ and $C_M(\alpha)$. The subroutine package is intended for use with a blade-element/momentum code where track of a specific airfoil table for each blade element is required. Airfoil data are therefore given for one or for more airfoils.

These airfoil data tables are stored in the common area `/profidata996/`

Each airfoil is associated with one table.

Data for at least one airfoil is needed.

Static airfoil data, $\alpha$, $C_{L,stat}$, $C_{D,stat}$ and $C_{M,stat}$ are stored in variables `alfastin`, `clstin`, `cdstin` and `cmstin`.

For calculation with the dynamic stall model ($lcncl=2$), the static data for $C_L$ is however not used from the $C_{L,stat}$ variable, but from the variables $f_{stat}$, $C_{L,inv}$ and $C_{L,sep}$ as explained in section 3.3.

If the Øye $f(\alpha)$ relationship is used ($LFMETH=2$), then the input airfoil data tables should include tabulated values of $f_{stat}$, $C_{L,inv}$ and $C_{L,sep}$ as input. The tabulated values of $C_{L,stat}$ are then ignored.

If the modified Kirchoff flow $f(\alpha)$ relationship is used ($LFMETH=2$), the same also holds. The tables of $f_{stat}$, $C_{L,inv}$ and $C_{L,sep}$ could, however, then automatically be generated by a call to the subroutine `kirchmake_c`. The subroutine `kirchmake_c` fills the tables `fnstin`, `cnvinvin` and `cnsepin` ($f_{stat}$, $C_{L,inv}$ and $C_{L,sep}$).
Airfoil data are stored in matrices $\text{alfastin}(ia,ip)$, $\text{clstsin}(ia,ip)$ etc. The second index refers to a table number, i.e. which airfoil it is. The first index refers to the row number (angle of attack index) in the airfoil data table.

Apart from matrices $\text{alfastin}$, $\text{clstsin}$, $\text{cdstsin}$, $\text{cmstsin}$, $\text{fnstsin}$, $\text{cninvin}$ and $\text{cnsepin}$, data for the zero lift angle of attack lift curve slope is also needed. The zero lift angle of attack and the lift curve slope are stored in the variables $\text{alfa0}$ and $\text{cn}_\text{alfa}$. ($\text{cn}_\text{alfa}$ should be in units $1/\text{radians}$ and $\text{alfa0}$ in radians).

The zero lift angle of attack and the lift curve slope around $\pm 180^\circ$ angle of attack is also needed as input if the angle of attack exceeds $\pm 90^\circ$. These are stored in variables $\text{cn}_\text{alfa2}$ and $\text{alfa02}$.

Furthermore the /profidata996/ area contains $\text{cn1pos}$ and $\text{cn1neg}$ which is the critical $C_N$ conditions needed for the vortex lift calculations if the method $\text{LVORMETH}=1$ is used (see section 3.3.4).

An example of one airfoil data table is shown below\(^2\)

<table>
<thead>
<tr>
<th>Alfa</th>
<th>Cl</th>
<th>Cd</th>
<th>Cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10.0</td>
<td>-0.8600</td>
<td>0.0150</td>
<td>-0.0375</td>
</tr>
<tr>
<td>-4.0</td>
<td>-0.1283</td>
<td>0.0067</td>
<td>-0.0785</td>
</tr>
<tr>
<td>-2.0</td>
<td>0.1252</td>
<td>0.0068</td>
<td>-0.0834</td>
</tr>
<tr>
<td>2.0</td>
<td>0.6273</td>
<td>0.0073</td>
<td>-0.0925</td>
</tr>
<tr>
<td>4.0</td>
<td>0.8748</td>
<td>0.0079</td>
<td>-0.0964</td>
</tr>
<tr>
<td>6.0</td>
<td>1.1190</td>
<td>0.0086</td>
<td>-0.0997</td>
</tr>
<tr>
<td>8.0</td>
<td>1.3525</td>
<td>0.0102</td>
<td>-0.1014</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5349</td>
<td>0.0154</td>
<td>-0.0973</td>
</tr>
<tr>
<td>12.0</td>
<td>1.5899</td>
<td>0.0271</td>
<td>-0.0843</td>
</tr>
</tbody>
</table>

\(^2\) Note that the angle of attack as input to $\text{alfastin}(.,.)$ should be in radians and not degrees.
This table contains 11 rows. If it would refer to airfoil number 1, then the 11 $C_L$-values should be stored in $\text{clstin}(i,1)$ with $i$ from 1-11.3

Additional to $\text{alfastin}$, $\text{clstin}$, $\text{cdstin}$, $\text{cmstin}$, $\text{fnstin}$, $\text{cninvin}$ and $\text{cnsepin}$ the variable $\text{nin}(ip)$ (in the same common area as $\text{alfastin}$ et.c.) is also needed. $\text{nin}$ tells how many rows that are used in each table. In the example above, $\text{nin}(1)=11$ should be set.

4.5.1.1 Range of angle of attack in input airfoil tables.

The range of angles of attack in each airfoil data table must cover the angles of attack that will be encountered during the calculations. If an angle of attack is calculated outside the input range, then the program will halt (in subroutine interp999) when $C_L$, $C_D$ and $C_M$ values are sought.

To cover the full 360 degrees range of angles of attack, $\alpha$ should be given in the range [-180,180] degrees.

4.5.1.2 Lift curve slope and zero lift angle of attack

The zero lift angle of attack and the lift curve slope are stored in the variables $\text{alfa0}$, $\text{cn_alfa}$, $\text{cn_alfa2}$ and $\text{alfa02}$.

The zero lift angle of attack and the lift curve slope are however input to the subroutine $\text{dyncl_c}$ as formal parameters together with the variable $\alpha_{\text{case}} (\text{acase})$ as explained in section 3.6.

The variables $\text{alfa0}_\text{used}$ and $\text{clalfa}_\text{used}$ should be set to the appropriate values of $\text{alfa0}$ $\text{cn_alfa}$, $\text{cn_alfa2}$ and $\text{alfa02}$.

The table must of course include the full range of angles of attack that will be encountered during the calculations. If an angle of attack is calculated to e.g. 20 degrees and the table with $\text{alfa}[-10,16]$ is used, then the program will stop.
alfa02. None of the variables alfa0, cnalfa, cnalfa2 and alfa02 is, however, actually used in the subroutine dyncl_c or subroutines directly called by dyncl_c.

alfa0_used and clalfa_used should be taken as the zero lift angle of attack and the lift curve slope for the cninv curve, \(C_{L, \text{inv}}(\alpha)\)-curve.

### 4.5.2 Choices between submodels

Variables to store choices for which sub-model to use for the dynamic stall calculations are stored in the common-area /typeparam/. These variables need to be set outside of the dyncl_c subroutine.

- **Lcncl**
  This parameter should be set to 2 if the dynamic stall model is used. Lcncl=1 was an old, now removed method.

- **Lpotmeth**
  Lpotmeth should be set to 3 or 4 to obtain methods as explained in section 3.2.

- **Lfmeth**
  Lfmeth should be set to 2 or 4 to obtain methods as explained in section 3.3.2.

- **Lvormeth**
  Lvormeth should be set to 2 or 1 to obtain methods as explained in section 3.3.3 or 3.3.4

- **Lcddyn**
  Lcddyn should be set to 0, 1 or 2. If lcdyn=0, then \(C_D\) will be returned as static \(C_D\).
  The choices between lcdyn=1 and lcdyn=2 are explained in section 3.4

- **Ldut**
  If ldut=0, then the effect of a varying velocity on the separation point position is neglected. The same effect is also obtained by setting \(f_u\) to zero.
4.5.3 Coefficients and time constants

The area /potcoeff/ contains the coefficients for the inviscid circulatory lift response.

/tfparam/ and /vorparam/ contain input variables for the separation lag part and the vortex lift part of the dynamic stall model.

/tfparam/
- **tp** Time constant $T_p$. See section 3.3.1
- **tf_in** Time constant $T_f$. See section 3.3.2
- **fpar_i** and **fpar_d** are not used any more.
- **fcrit** is not used any more
- **fufac** Constant $f_u$, see section 3.3.1
- **acd** Constant $A_d$, see section 3.4

/vorparam/
- **tvo** Time constant $T_v$. See section 3.3.3
- **tvl** Time constant used with lvormeth=1. See section 3.3.4 and the code
- **tvs** Time constant used with lvormeth=1. See section 3.3.4 and the code

4.5.4 Variables used for state variables and other variables needed for internal communication

The areas /curval/ and /oldval/ contain variables used for the calculation of the dynamic stall (state variables). Values at the current time step is stored in the variables in /curval/ and the values at the previous time step is stored in /oldval/. None of the variables in /curval/ or /oldval/ need to be set outside of the dyncl_c subroutine package except for that start-conditions for the /oldval/-variables must be set as explained in section 4.1.
4.5.5 Variables in common areas that must be set at each time step outside of the DYNSTALL-package

The two variables \( v_{rel} \) and \( u_p \) in the common area /curval/ must be set at each time step. \( v_{rel}(ib,ir) \) is \( V_{rel} \) for blade element IB and radial position IR (see section 3.2.1). \( u_p(ib,ir) \) is the plunging velocity \( u_p \) (see section 3.2.2).

4.6 Generation of FNSTIN, CNINVIN and CNSEPIN tables

The tables of \( f_{stat}(\alpha) \), \( C_{L,inv}(\alpha) \) and \( C_{L,sep}(\alpha) \) are needed for the calculation of a dynamic \( C_L \).

If the Øye \( f(\alpha) \) relationship is used (LFMETH=2), then the airfoil data for \( f_{stat}, C_{L,inv} \) and \( C_{L,sep} \) must be included in input files. Values of alfa0_used and clalfa_used should then be taken as the zero lift angle of attack and the lift curve slope of the \( cninvin \) curve. **Note that the Øye method, in the current implementation, only works for the ±90° range of angles of attack.**

If the modified Kirchoff flow \( f(\alpha) \) relationship (lfmeth=4) is used, tables of \( f_{stat}, C_{L,inv} \) and \( C_{L,sep} \) can be automatically generated by a call to the subroutine kirchmake_c. The subroutine kirchmake_c fills the tables fnstin, cninvin and cnsepin (\( f_{stat}, C_{L,inv} \) and \( C_{L,sep} \)).

In the AERFORCE package, kirchmake_c is called at the first time step to set up \( f_{stat}, C_{L,inv} \) and \( C_{L,sep} \)-tables if lfmeth=4.

The subroutine kirchmake_c also fills the variables of alfa0 cn_alfa, cn_alfa2 and alfa02 from which appropriate values of alfa0_used and clalfa_used can be selected.
5 Sub-methods and values for semi-empiric constants

Reported use of the FFA dynamic stall model is the use in the Stallvib project [3], [11].

The model is also used in a sensitivity analysis [14].

5.1 An optimization study to find optimum values

Optimal parameters for the dynamic stall model was sought by optimization in [15], [16].

In this work experimental data from several sources were used. Values for the semi-empirical constants in the dynamic stall model that would minimize the difference between simulation and experimental results were sought.

In [15] and [16] the Kirchoff flow model was used.

The result was that there is a substantial span in the result of optimum values for the parameters for different airfoils and different cases.

**$T_v$ and $T_f$**

One conclusion is that with $T_v \approx 2$ and $T_f \approx 5$, a reasonable good agreement was obtained for a large number of cases.

**$A_{cd}$**

Optimum values semi-empirical parameter $A_{cd}$ was found in the range from 0 to 0.43. Most values were below 0.1. A value of $A_{cd} \approx 0.05$ to $A_{cd} \approx 0.1$ seams reasonable to use.

**$T_p$**
The value for $T_p = 0.8$ was used in most cases in the optimization study [16]. An optimum value was also sought, but the span of optimum values was substantial so that it is hard to recommend a value based on this study. The value $T_p = 0.8$ was taken from [10] for plunging motion.

5.2 lcddyn

In calculations for the stallvib project it was found that $lcddyn=2$ gave better agreement with experimental data than using $lcddyn=1$.

5.3 Effects of varying velocity on the separation point

The model to account for changes in the unsteady pressure gradient due to a varying free stream velocity was introduced during the STALLVIB project. Very limited investigations [11] found a value of the semi-empiric constant $f_u = 0.5$ to be reasonable.

$f_u$ should, however, likely be a function of angle of attack or a function of the separation point position. At high angels of attack, a value of $f_u = 0.5$ might be too high.

5.4 Recommended values of submodels and semi-empirical parameters

The dynamic stall model captures several features of unsteady airfoil aerodynamics and dynamic stall. The model is often able to quite well predict the $C_L(t)$ and $C_D(t)$ behavior. The model is, however, not a solution of the flow for the dynamic stall process. Different airfoils might require different values of semi-empirical constants to get a good fit to the real dynamic stall behavior.

Based on experience with the semi-empirical dynamic stall model and based on the experience from the optimization study [16] it
seems a bit unwise to recommend values for the semi-empirical constants for a “general airfoil”. Nevertheless, such values will be asked for so why not give a best guess.

For use in aeroelastic calculations with “common” wind turbine blade airfoils the author’s current default setting is:

\[ T_p = 0, \quad T_f = 5, \quad T_v = 2, \quad A_{cd} = 0.08, \quad f_u = 0.25 \]

The choice of sub-models should then be:

\[ lcncl = 2, \quad lpotmeth = 4, \quad lfmeth = 4, \quad lvormeth = 2, \quad ldddyn = 2, \quad ldut = 1 \]

### 5.4.1 Effect on aerodynamic damping, fatigue and extreme loads

Sometimes it is good to be a bit on the conservative side in load calculations. How then do the values of the semi-empirical constants affect loads? Unfortunately that is hard to say in general. Some rules of thumb could, however, be given.

#### 5.4.1.1 Damping of flap-wise vibrations in stall

- Increase in \( T_p \) => increase
- Increase in \( T_f \) => increase
- Increase in \( T_v \) => increase
- \( A_{cd} \) hardly affects the flap-wise damping
- \( f_u \) does not affect the flap-wise damping
- \( lpotmeth = 4 \) instead of \( lpotmeth = 3 \) does not make a big change

#### 5.4.1.2 Damping of edge-wise vibrations in stall

An increase in damping here means more positive or less negative damping.

- Increase in \( T_p \) => increase
- Increase in \( T_f \) => increase
- \( T_v \) hardly affects the edge-wise damping
Increase in $A_{cd}$ => increase
Increase of $f_u$ => decrease
lpotmeth=4 instead of lpotmeth=3 increases the damping.

5.4.1.3 Extreme loads

If the extreme loads increase with increasing maximum values of $C_L$, then following holds.

Increase in $T_p$ => increase
Increase in $T_f$ => increase
Increase in $T_v$ => increase
$A_{cd}$ hard to say
$f_u$ hard to say
lpotmeth=4 instead of lpotmeth=3: hard to say
References


**Title**
DYNSTALL: Subroutine Package with a Dynamic stall model.

**Author**
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**Checked by**
Björn Montgomerie

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**Abstract**
A subroutine package, called DYNSTALL, for the calculation of 2D unsteady airfoil aerodynamics is described. The subroutines are written in FORTRAN.

DYNSTALL is a basically an implementation of the Beddoes-Leishman dynamic stall model. This model is a semi-empirical model for dynamic stall. It includes, however, also models for attached flow unsteady aerodynamics. It is complete in the sense that it treats attached flow as well as separated flow. Semi-empirical means that the model relies on empirically determined constants. Semi because the constants are constants in equations with some physical interpretation. It requires the input of 2D airfoil aerodynamic data via tables as function of angle of attack.

The method is intended for use in an aeroelastic code with the aerodynamics solved by blade/element method.

DYNSTALL was written to work for any 2D angles of attack relative to the airfoil, e.g. flow from the rear of an airfoil.

**Keywords/Nyckelord**
Aerodynamics, Wind Energy, Unsteady Aerodynamics, Dynamic Stall, Wind Turbine

**Distribution**