

# CHALMERS



## Fatigue Assessment of Concrete Foundations for Wind Power Plants

*Master of Science Thesis in the Master's Programme Structural Engineering and  
Building Performance Design*

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Department of Civil and Environmental Engineering  
Division of Structural Engineering  
Concrete Structures  
CHALMERS UNIVERSITY OF TECHNOLOGY  
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Examensarbete / Institutionen för bygg- och miljöteknik,  
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## ABSTRACT

The demands on cleaner and renewable energy have increased with the depletion of natural resources and a way to meet these demands is to use wind power. With an increasing amount of wind power plants the design of foundation slabs for these are of interest. The foundation slab for a wind power plant is subjected to cyclic loading from the wind and there have been uncertainties concerning the effect of fatigue. From the beginning of 2011 all new structures in Sweden should be designed according to Eurocode and as a result of this a new approach to calculate the influence of fatigue is to be considered.

The aim of this project is to give recommendations for designing concrete foundation slabs with regard to fatigue. This is done by clarifying the issues of fatigue for a foundation slab for a wind power plant, comparing methods for fatigue assessment according to Eurocode 2 and fib Model Code 2010 as well as performing parametric studies of the fatigue design of a foundation slab and identifying parameters that have significant impact on fatigue of a foundation slab for a wind power plant.

When performing a fatigue assessment the choice of assessment method is important. In this project two fatigue assessment methods regarding reinforcement, according to Eurocode 2, have been included. One of the methods takes the frequency of the load into account and uses the Palmgren-Miner damage summation to estimate the fatigue damage, while the other method uses an equivalent value for the fatigue load and a reference number of cycles. For fatigue assessment of compressed concrete two methods according to Eurocode 2 have been used, as well as a method according to fib Model Code 2010. One of the Eurocode methods and the fib Model Code method are damage calculation methods, while the second Eurocode method just checks for an adequate fatigue life by using the maximum and minimum compressive stress under the frequent load combination.

The fatigue assessment and parametric study performed shows that the dimensions of the slab have an impact on fatigue of the foundation slab. It is concluded that the damage calculation methods are preferable when performing a fatigue assessment based on a spectrum load, like the wind acting on a wind power plant. The fatigue assessment methods that do not take the frequency of the load into account has been seen to not be valid for such a complex load used in this project.

Key words: Fatigue, fatigue assessment method, wind power plant, concrete foundation slab, Eurocode 2, fib Model Code 2010

# Utmattningsbedömning av Betongfundament för Vindkraftverk

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## SAMMANFATTNING

Kraven på renare och förnybar energi har ökat med sinande naturresurser och ett sätt att möta dessa krav är att använda vindkraft. Med ett ökande antal vindkraftverk har designen av fundament för dessa blivit mer av intresse. Ett vindkraftverks fundament är utsatt för cyklisk last från vinden och det har varit osäkerheter angående effekten av utmattning. Från början av 2011 ska alla nya byggnader i Sverige designas enligt Eurocode och i och med detta ska nya metoder för beräkning av utmattning användas.

Syftet med det här projektet är att ge rekommendationer för design av betongfundament med avseende på utmattning. Detta görs genom att klargöra utmattningsproblematiken för ett fundament för ett vindkraftverk, jämföra metoder för utmattningsverifiering enligt Eurocode 2 och fib Model Code 2010 samt att utföra parameterstudier avseende utmattningsdesign av ett fundament och att identifiera parametrar som har en betydande inverkan på utmattning av ett fundament för ett vindkraftverk.

Vid utmattningsverifiering är valet av metod viktigt. I det här projektet har två metoder enligt Eurocode 2 avseende armeringsutmattning använts. En av metoderna tar hänsyn till lastfrekvens och använder Palmgren-Miners delskadesummering för att uppskatta utmattningsskadan, medan den andra metoden använder ett ekvivalent värde för utmattningslasten och ett referensvärde för antalet lastcykler. För utmattningsverifiering av tryckt betong har två metoder enligt Eurocode 2 använts, samt en metod enligt fib Model Code 2010. En av Eurocode-metoderna och metoden från fib Model Code använder delskadesummeringen, medan den andra Eurocode-metoden kontrollerar att livslängden med avseende på utmattning är tillräcklig genom att använda den maximala och minimala tryckspänningen under den frekventa lastkombinationen.

Utmattningsverifieringen och parameterstudien som har utförts visar att fundamentets dimensioner har en inverkan på utmattning av fundamentet. En slutsats som har dragits är att metoderna som använder delskadesummering är att föredra när en utmattningsverifiering av en spektrumlast utförs. De metoder som inte tar hänsyn till lastfrekvens har visat sig vara otillfredsställande för den komplexa last som använts i detta projekt.

Nyckelord: Utmattning, utmattningsverifiering, vindkraftverk, betongfundament, Eurocode 2, fib Model Code 2010

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# Preface

This study was initiated by Ramböll due to uncertainties regarding the effect of fatigue on a foundation slab for a wind power plant. The project was carried out between January and August 2011 in cooperation with Ramböll and the Department of Structural Engineering, Concrete Structures, Chalmers University of Technology, Sweden.

We would like to thank Ramböll Byggt teknik, Göteborg, for the opportunity to work with this thesis and especially our supervisor, Per Lindberg, for his support and for providing us with information during our work.

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Without the help of Karl Lundstedt at Skanska Teknik, Malmö, this project would not have turned out as it did and we deeply appreciate his commitment and time. We are also grateful to Peikko Sverige AB and Siemens for providing us with material during the work with this project.

Last but not least, we would like to show our appreciations to our opponents Malin Johansson and Terese Löfberg for always being there for us with comments and coffee breaks.

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Frida Göransson & Anna Nordenmark

# Notations

## Roman upper case letters

$A_{c0}$	is the area the horizontal compressive force is acting on
$A_s$	is the reinforcement amount
$A_s$	is the required reinforcement amount
$A_s$	is the amount of bottom reinforcement
$A_{sv}$	is the amount of shear reinforcement
$A'_s$	is the amount of top reinforcement
$C_{max}$	is the maximum compressive concrete force in a cycle
$C_{min}$	is the minimum compressive concrete force in a cycle
$D$	is the bending diameter of a bent bar
$D$	is the fatigue damage
$D_{Ed}$	is the damage caused by the stress range
$E_{cd,max,equ}$	is the maximum compressive stress level
$E_{cd,max_i}$	is the maximum compressive stress level
$E_{cd,min,equ}$	is the minimum compressive stress level
$F_C$	is the compressive force in the force couple
$F_T$	is the tensile force in the force couple
$F_{res}$	is the horizontal force
$F_z$	is the normal force transferred from the tower to the slab
$G$	is the self weight of the foundation slab
$M_{fat.amplitude}$	is the peak to peak fatigue moment
$M_{res}$	is the overturning moment
$M_z$	is the torsion moment
$N$	is the number of load cycles until fatigue failure
$N(\Delta\sigma_i)$	is the ultimate number of cycles for stress range $\Delta\sigma_i$
$N^*$	is a reference value for number of cycles until fatigue failure depending on reinforcement type
$P$	is the horizontal soil pressure
$R_{eq_i}$	is the stress ratio
$R_{equ}$	is the stress ratio
$R_{res}$	is the reaction force from the ground
$S_{c,max}$	is the maximum compressive stress level
$S_{c,min}$	is the minimum compressive stress level
$T$	is the tensile force estimated in the strut and tie model
$T_{max}$	is the maximum tensile force in the reinforcement in a cycle
$T_{min}$	is the minimum tensile force in the reinforcement in a cycle

### Roman lower case letters

$b$	is the distance from the edge of the slab to the gravity centre of the reaction force
$f_{cd}$	is the design compressive concrete strength in [MPa]
$f_{cd,fat}$	is the design fatigue resistance for concrete
$f_{ck}$	is the characteristic compressive strength in [MPa]
$f_{ck,fat}$	is the reference fatigue compressive strength
$f_{ctk0}$	is 10 MPa
$f_{yd}$	is the design yield strength of the reinforcement
$k_4$	is a parameter influencing the design strength for concrete struts
$k_1$	is a coefficient affecting the fatigue strength
$k_1$	is the exponent that defines the slope of the first part of the S-N curve
$k_2$	is the exponent that defines the slope of the second part of the S-N curve
$n_{Ri}$	is the number of cycles causing failure at the same stress level and stress range
$n_{Si}$	is the number of acting stress cycles at a given stress level and stress range
$n(\Delta\sigma_i)$	is the number of cycles for stress range $\Delta\sigma_i$
$s$	is a coefficient depending on the type of cement
$t_0$	is the concrete age in days when first subjected to fatigue loading

### Greek upper case letters

$\Delta M$	is half the amplitude of the fatigue moment
$\Delta S_c$	is the stress range
$\Delta\sigma_{Rsk}$	is the reference fatigue stress range after $N^*$ cycles
$\Delta\sigma_{Rsk}(N^*)$	is the reference resisting fatigue stress range at $N^*$ cycles
$\Delta\sigma_{S,max}$	is the maximum steel stress range

### Greek lower case letters

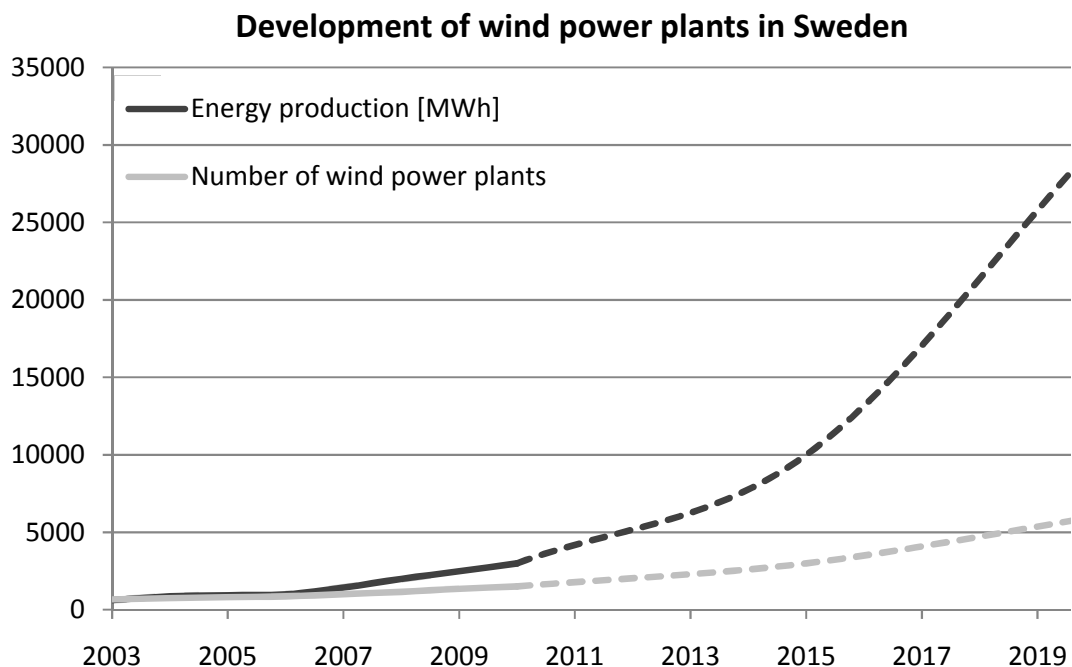
$\beta_{cc}(t_0)$	is a coefficient for concrete compressive strength at first load application
$\beta_{c,sus}(t, t_0)$	is a coefficient that takes the effect of high mean stresses during loading into account
$\phi$	is the diameter of the reinforcement bar
$\gamma_C$	is a partial factor for concrete
$\gamma_f$	is a partial factor for loads, according to IEC 61400-1
$\gamma_{F,fat}$	is a partial factor for fatigue loading
$\gamma_G$	is a partial factor for permanent loads
$\gamma_m$	is a partial factor for materials, according to IEC 61400-1

$\gamma_n$	is a partial factor for consequence of failure, according to IEC 61400-1
$\gamma_Q$	is a partial factor for variable loads
$\gamma_s$	is a partial factor for steel
$\gamma_{s,fat}$	is a partial factor for fatigue that takes the material uncertainties into account
$\nu$	is a parameter influencing the design strength for concrete struts
$\sigma_{c0,max}$	is the maximum compressive concrete stress in a cycle
$\sigma_{c0,min}$	is the minimum compressive concrete stress in a cycle
$\sigma_{cd,max,equ}$	is the maximum equivalent compressive stress for $10^6$ cycles
$\sigma_{cd,min,equ}$	is the minimum equivalent compressive stress for $10^6$ cycles
$\sigma_{c,max}$	is the maximum compressive stress in a cycle
$\sigma_{c,max}$	is the maximum concrete compressive stress under the frequent load combination
$\sigma_{c,min}$	is the minimum compressive stress in a cycle
$\sigma_{c,min}$	is the minimum concrete compressive stress in the section where $\sigma_{c,max}$ is found
$\sigma_{Rd,max}$	is the largest stress that can be applied at the edge of compression node in strut and tie models
$\sigma_{s,max}$	is the maximum tensile steel stress in a cycle
$\sigma_{s,min}$	is the minimum tensile steel stress in a cycle
$\xi$	is a reduction factor for the reference fatigue stress range for bent bars

# 1 Introduction

## 1.1 Background

The demands on cleaner and renewable energy have increased with the depletion of natural resources and a way to meet these demands is to use wind power. Between 1999 and 2010 the energy production from wind power plants in Sweden increased extensively as a result of this, Energimyndigheten (2011). Year 2010 the energy production from wind power plants was 3,0 TWh and the Swedish government has set a goal of an increase of the energy production from wind power to 30,0 TWh year 2020. This would result in a need for 3000-6000 wind power plants instead of today's 1500, see Figure 1.1.



*Figure 1.1 The energy production from wind power and number of wind power plants in Sweden 2003-2010 and the planning goal for 2020. Based on tables from Energimyndigheten.*

The technology for wind power plants is developing fast and plants become bigger and more effective. This leads to that the lifetime of a wind power plant is short, but the foundations can be given a longer life span.

The foundation slab for a wind power plant is subjected to cyclic loading from the wind. In earlier design there have been uncertainties concerning the effect of fatigue and this has in many cases been assumed ignorable. Now with design according to Eurocode and larger wind power plants the influence of fatigue has become more of interest.

From the beginning of 2011 all new structures in Sweden should be designed according to Eurocode. This European Standard is a design code with purpose to simplify the cooperation between engineers from different European countries and to function as a legal document for the engineer to relate to. As a result of this code a new approach to calculate the influence of fatigue is to be considered.

## **1.2 Aim and objectives**

The aim of this project is to give recommendations for designing concrete foundation slabs with regard to fatigue. Objectives of this project are to:

- Clarify the issues of fatigue for a foundation slab for a wind power plant in relation to the general design problems.
- Compare methods for fatigue assessment according to Eurocode 2 and fib Model Code 2010, as well as their backgrounds.
- Perform parametric studies of the fatigue design of a foundation slab.
- Identify parameters that have significant impact on the fatigue of a foundation slab for a wind power plant.
- Formulate recommendations for fatigue design of foundation slabs.

## **1.3 Method**

To increase the knowledge of fatigue a literature study was done. This study included fatigue of reinforced concrete as well as concrete and reinforcement behaviour under fatigue loading. The approaches of fatigue according to Eurocode 2 and fib Model Code 2010 was also part of this study. Further a literature study concerning the design of foundation slabs for wind power plants has been carried out.

A foundation slab was designed for initial conditions disregarding the fatigue loads. This slab was then investigated with regard to fatigue using both Eurocode 2 and fib Model Code 2010 to see how and if fatigue influences the design. A comparison between the approaches was done.

When the design of the slab was made, the width and height of the slab was varied within reasonable limits. To do this each parameter was altered to see the influence it has on the foundation slab with regard to fatigue. During the change of parameters the static design was always verified.

After the study of the behaviour of the foundation slab with respect to fatigue, an analysis of the results from the design of the slab and the fatigue assessment was done.

## 1.4 Limitations

Primarily the foundation slab for a wind power plant was investigated but the results should be possible to apply on slabs in machine rooms. However the calculations were only performed for foundation slabs for wind power plants.

The project was limited to investigating onshore foundations that are square and with a constant height. The most common type of foundations for onshore wind power plants are gravity foundations, while this is the type studied. The transfer of loads between the tower and the slab is done by an inserted anchor ring in the concrete. The loads taken into consideration in the study were the self weight of the slab and the loads transferred from the tower to the slab. These loads consist of normal and horizontal force, and overturning and torsion moment.

The fatigue assessment regarding concrete only included a verification of the concrete fatigue compressive strength since this enabled a comparison between the methods in fib Model Code 2010 and Eurocode 2. For the fatigue verification of reinforcement the methods used was according to Eurocode 2 and will only verify the fatigue tensile strength of the reinforcement.

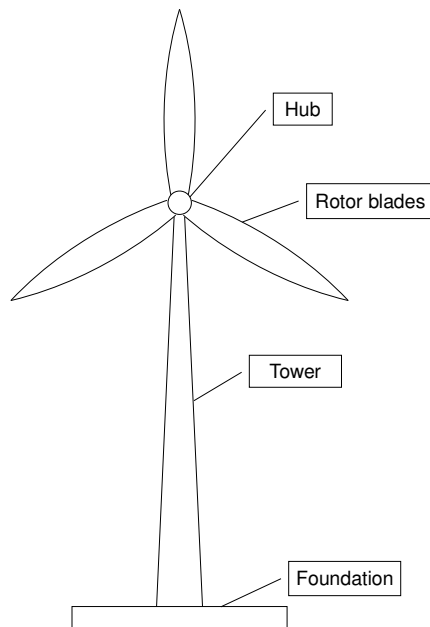
## 2 Foundation Slabs for Wind Power Plants

### 2.1 Ground conditions

Wind power plants are used at locations that fulfil requirements for wind conditions. Every location is different with regard to the ground conditions and may require different types of foundations. If a wind power plant is built on rock, a special concrete rock adapter can be used for anchorage. For clay there is often a need to use piled foundations due to the low bearing capacity of clay. The most common foundation type for wind power plants is gravity foundations which are suitable for soil types from sand and clay to hard rock. When deciding on the geometry of the slab the ground conditions are an influencing factor since it is crucial that the ground can resist the pressure under the slab. The foundation and soil must also have a sufficient rotational stiffness which is estimated as a combined stiffness for foundation and soil.

### 2.2 Loading conditions

A wind power plant is subjected to a range of different actions. Many of the loads are transferred from the wind power plant tower to the foundation slab. The loads can be divided into two categories, static loads and fatigue loads. The loads consist for example of the self weight of the plant, the wind load acting on the turbine at hub-height and a sectional overturning moment on the foundation slab that the wind load gives rise to. Figure 2.1 shows the different parts of a wind power plant.



*Figure 2.1 The different parts of a wind power plant.*



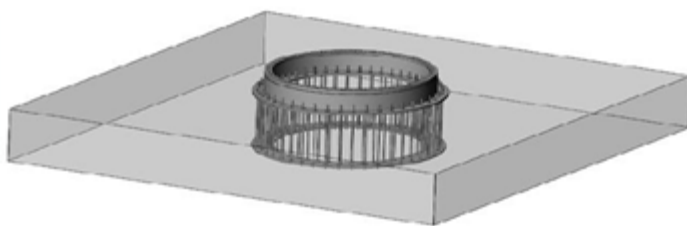
The slab is also subjected to other variable actions like snow and temperature. These actions are though small in comparison to the others. The self weight of the whole structure, the wind power plant and the foundation slab, is resisted by the resulting earth pressure acting on the foundation slab.

The foundation slab is subjected to a fatigue load caused by the wind acting on the tower and the movement of the turbine. This fatigue load is more complex and differs from the fatigue load caused by traffic on a bridge for example. The wind is acting on the tower with varying speed and direction. When designing the foundation slab the fatigue loads are often given by the supplier of the wind power plant and may differ between different suppliers. The height and size of the tower are also influencing factors for the fatigue load acting on the slab. Fatigue loads specified by the supplier are often given in form of tables from which fatigue damage can be calculated.

## 2.3 Typical foundation slab designs

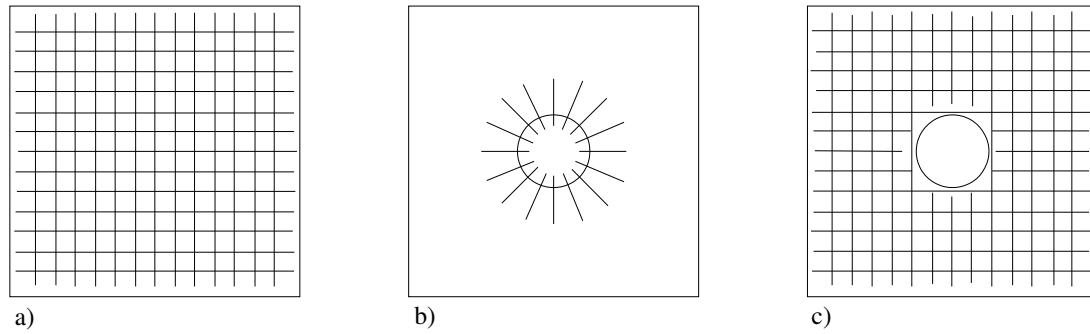
The foundation slab for a wind power plant is usually square, circular or octagonal. The top of the slab is either flat or with a small slope of maximum 1:5. This slope is a way of making the slab more economically beneficial due to the fact that less concrete is needed. It also ensures that water is led away from the slab. The maximum slope is chosen so that no upper formwork has to be used when casting the concrete and the maximum slab thickness is provided where the highest shear force and moment are acting. The concrete strength class for foundation slabs of wind power plants normally ranges from C30/37 to C35/45.

The size of the foundation slab is determined by the demand on foundation stiffness, set by the manufacturer of the wind power plant. This is to avoid self-oscillation and limit the risk of settlements. Due to this demand the foundation slabs normally have a width of 15 to 20 meters and a thickness of 1.5 to 2.5 meters. The manufacturer also provides information about an anchor ring that is used to anchor the tower in the foundation slab. For a principal drawing of an anchor ring see Figure 2.2.



*Figure 2.2 Square foundation slab of a wind power plant with anchor ring, Peikko Sverige AB.*

Different foundation slab shapes result in differences in the resulting earth pressure and different reinforcement layouts. For square and octagonal slabs the bottom reinforcement is placed in two directions, perpendicular, while the top reinforcement is radial through the anchor ring and similar to the bottom reinforcement outside the ring, see Figure 2.3. For circular slabs both the top and bottom reinforcement can be radial.



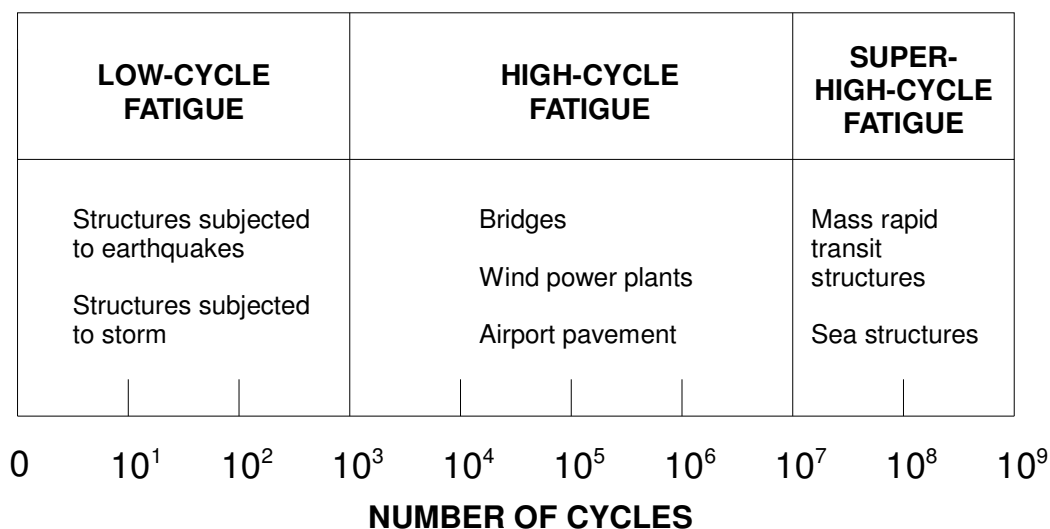
*Figure 2.3 Reinforcement layouts for a square foundation slab of a wind power plant: (a) Bottom reinforcement (b) Radial top reinforcement through the anchor ring (c) Top reinforcement outside the anchor ring.*

## 3 Fatigue of Reinforced Concrete

### 3.1 General

When a structure is subjected to a cyclic load, failure can occur before the static loading strength of the material is reached. This type of failure is known as a fatigue failure. Typical cyclic loads are wind, waves, traffic loads and vibrations from machines. These affect structures like bridges, tall buildings, offshore structures and wind power plants. The fatigue capacity of the material is influenced by several different parameters such as load frequency, maximum load level, stress amplitude and material composition.

Cyclic load, or fatigue loading, is divided into three different categories: low-cycle, high-cycle and super-high-cycle fatigue. The number of load cycles determines the type of fatigue. Few load cycles, up to  $10^3$  cycles, give low-cycle fatigue. If the number of cycles is between  $10^3$ - $10^7$  it is referred to as high-cycle fatigue. The last category is super-high-cycle fatigue and it is with more than  $10^7$  cycles. See Figure 3.1 for examples of structures subjected to the different types of fatigue loading.



*Figure 3.1 Spectra of fatigue load categories and examples of structures subjected to the fatigue loads.*

When performing a fatigue life assessment of structural elements there are two basic approaches that can be used. One considers an analysis of crack propagation at the point under consideration and is based on linear elastic fracture mechanics. The second approach, which is more commonly applied, uses a curve that shows the relation between cyclic stress range and number of cycles to fatigue failure in logarithmic scales. This is known as a Wöhler or an  $S-N$  curve where  $S$  is the stress range and  $N$  is the number of cycles.

The  $S - N$  curves are derived using experimental data obtained from fatigue tests. The curve is presented by the “best fit” line with constant slope. In practice the curve used is parallel to that obtained from the fatigue tests, but with a deviation to achieve a safety margin, see Figure 3.2.

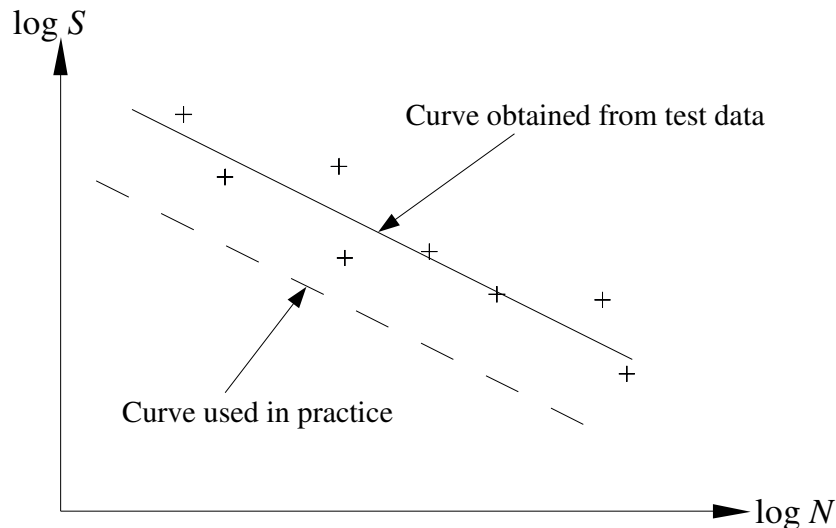


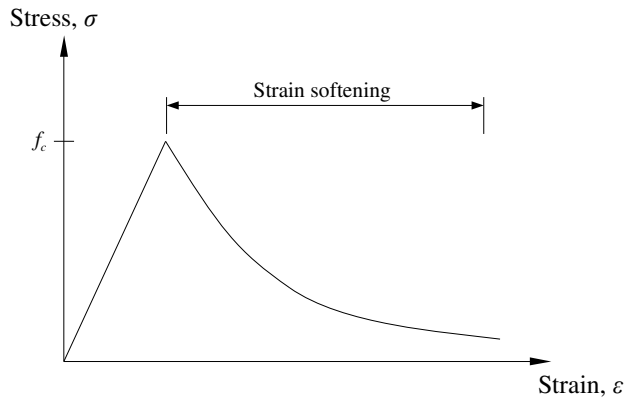
Figure 3.2 Principal  $S - N$  curve.

Fatigue failure is divided into three different stages: crack initiation, crack propagation and failure. However concrete and steel behave differently with regard to fatigue and reinforced concrete is treated as two separate materials when designing for this, Johansson (2004). Investigations have been made about how concrete behave and how reinforcement react, but only a few studies have been made of reinforced concrete members.

Due to the fact that the self weight of concrete is normally a large part of the total load, this counteracts the affect of fatigue. With the work towards more optimised structures the self weight is reduced and the utilization of the concrete strength is increased. This could lead to that fatigue will have a larger significance when designing in concrete.

## 3.2 Concrete

Concrete is not a homogenous material and during hardening of concrete pores and micro cracks are formed. It is also common that macro cracks are formed before any load is applied, this due to shrinkage and temperature differences. Because of the cracks and the inhomogeneity concrete can be regarded as a strain-softening material i.e. with large strains the stiffness decreases, see Figure 3.3.

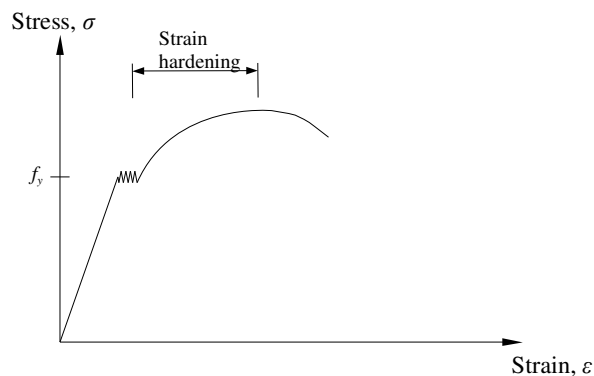


*Figure 3.3 Stress-strain relation for concrete showing the decreasing stiffness due to strain softening.*

When the concrete is subjected to a cyclic load there will be no clear crack initiation process, since cracks already exist. Instead these cracks will grow, slowly at first and then quicker, until failure in the remaining part of the concrete section, Svensk Byggtjänst (1994). Unlike steel, fatigue cracks in concrete have no identifiable surface topography and due to this fatigue failure in concrete structures is difficult to identify.

### 3.3 Reinforcing steel

In contrast to concrete steel is a strain-hardening material, which means that with large strains above yielding the strength increases, see Figure 3.4. This gives steel a fatigue stress limit and below this limit no fatigue failure will occur, Thun (2006).



*Figure 3.4 Stress-strain relation for steel showing the increasing strength due to strain hardening.*

For reinforcing steel cracks are initiated by discontinuities or stress concentrations in the bar; these cracks grow until they cause a brittle failure in the remaining uncracked part of the bar. It has been seen that the strength of reinforcement subjected to fatigue loading is reduced to about 44 % of the yield strength, CEB (1988).

The most important factor influencing the fatigue life of reinforcement is the stress amplitude. Other influencing factors are geometry (which affects stress concentrations in the bar) and environment (i.e. corrosion). With increasing bar diameter the fatigue strength of the reinforcement decreases. This is due to the increased risk of flaws in the surface because of the larger diameter/area. According to CEB (1988) bent bars, mechanical and welded connections and corrosion of the bars also result in a lowered fatigue performance of the reinforcement.

### 3.4 Fatigue of reinforced concrete members

If a reinforced concrete beam or slab is subjected to cyclic bending loading, cracks will appear and initial cracks will grow. With the crack propagation the tensile stresses in the concrete will redistribute to the reinforcement. Deformation of the member increases due to cyclic loading, which leads to that the importance of deformations, is generally greater than the actual fatigue life of the element.

Fatigue failure in reinforced concrete can occur in the concrete, reinforcement or in the connection between them. The failures are characterised as bending, which results in compression failure of the concrete or tensile failure of the reinforcement, shear or punching and bond failure.

Bending failure depends on how the concrete is reinforced. For under-reinforced sections the fatigue resistance is mainly determined by the fatigue properties of the reinforcement. Fatigue failure of the reinforcement due to tensile forces, caused by bending, occur without noticeable strain and is therefore difficult to predict. If the section is over-reinforced fatigue failure will occur in the compression zone in the concrete as a compression failure, CEB (1988).

Shear failure is depending on whether the beam or slab has shear reinforcement. For beams without shear reinforcement a critical shear crack may develop after only a few loading cycles. This shear crack appears when the tensile strength of the concrete is reached and the fatigue failure is determined by the propagation of this crack, CEB (1988). Two different fatigue failure modes for beams without shear reinforcement exist and are shown in Figure 3.5. The first mode is formation of a diagonal crack and the second is compression of the concrete at the top of the shear crack.

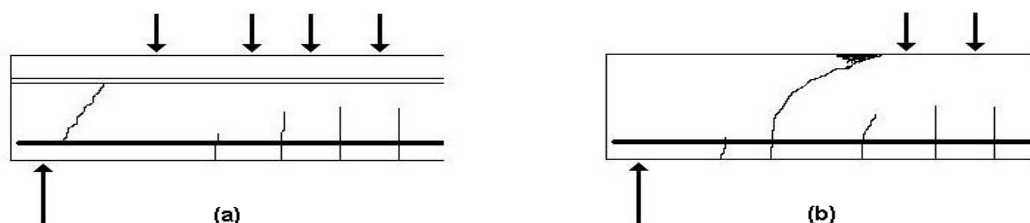
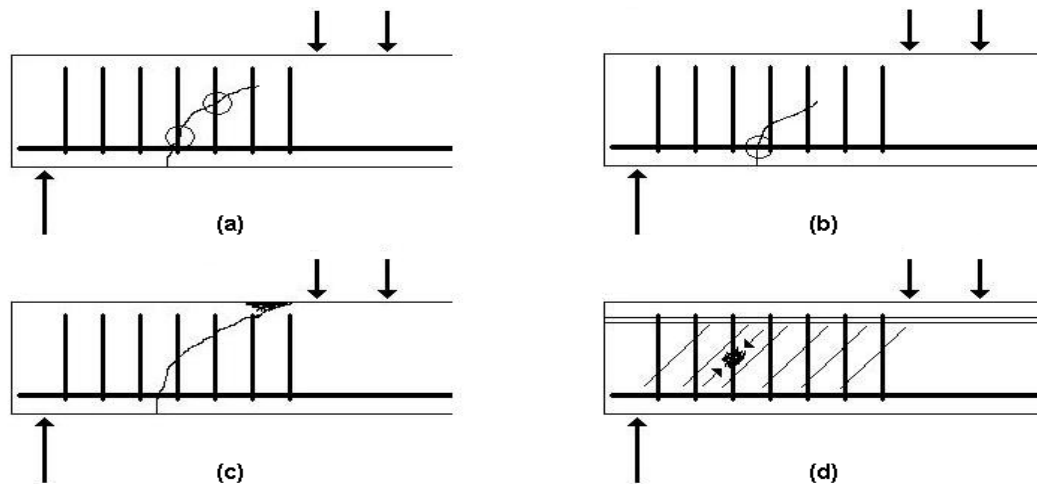


Figure 3.5 Possible shear fatigue failure modes in a beam without shear reinforcement: (a) diagonal cracking (b) compression of the concrete at the top of the shear crack

The fatigue behaviour of beams with shear reinforcement is depending on the fatigue properties of the reinforcement. In Figure 3.6 four fatigue failure modes are shown. The first two show when fatigue failure occur in the reinforcement, either in the stirrups or in the longitudinal reinforcement. The last two is when the concrete fails in compression, either at the top of the shear crack or in the middle of the beam between cracks.



*Figure 3.6 Possible shear fatigue failure modes in a beam with shear reinforcement: (a) fatigue of shear reinforcement (b) fatigue of longitudinal reinforcement crossing the shear crack (c) fatigue of the concrete in compression at the top of the shear crack (d) fatigue of the concrete in compression in the middle of the beam.*

Bond failure is failure between the concrete and the reinforcement. There are three different types of bond failures; splitting of the surrounding concrete, concrete failing in shear along the perimeter of the reinforcement bar and break down of the shear strength of chemical bonds between the reinforcement bar and the concrete. These failures are affected by e.g. load level, frequency of load, strength of concrete and confining effects, fib (2000).

Splitting of the surrounding concrete occur when the shear strength of the bond is sufficient and is caused by the radial pressure from the anchored reinforcing bar. This pressure develops when the principal tensile stress cracks the concrete and causes the bond forces to be directed outward from the bar. The effect of cyclic load on this type of failure is that under cyclic load the stress pattern changes due to stress redistribution. The repeated loading opens up longitudinal cracks and cyclic creep of concrete causes the stress pattern at ultimate failure to emerge earlier.

When the concrete has sufficient splitting resistance the fatigue bond failure will occur as shear failure of the concrete. This is the highest bond resistance and is determined by the shear strength of confined concrete.

Bond failure of chemical type of bonds has rarely been studied with regard to fatigue since chemical bonds are seldom used. However, when the concrete determines the strength of the chemical bond a reduction of the bond strength due to fatigue can be assumed, CEB (1988).

## 4 Fatigue Verification According to Eurocode 2 and fib Model Code 2010

### 4.1 Fatigue verification of concrete according to Eurocode 2

The fatigue verification methods for concrete in compression, according to Eurocode 2, are presented in EN 1992-1-1, SIS (2005). In Eurocode 2 there are two methods for fatigue verification of compressed concrete that can be applied on foundation slabs. These methods only take the compressive stresses in the concrete into consideration and not the number of load cycles until fatigue failure occurs. Instead a reference value of  $10^6$  load cycles is applied. The methodology used is that a comparison is done for the stresses caused by the cyclic load and a reference concrete strength for static load  $f_{cd,fat}$ . This reference concrete strength is defined by equation (4.5). According to the first fatigue verification method the compressed concrete has a sufficient fatigue resistance if the following condition is fulfilled:

$$E_{cd,max,eq} + 0,43\sqrt{1-R_{eq}} \leq 1 \quad (4.1)$$

Where:

$$R_{eq} = \frac{E_{cd,min,eq}}{E_{cd,max,eq}} \quad (4.2)$$

$$E_{cd,min,eq} = \frac{\sigma_{cd,min,eq}}{f_{cd,fat}} \quad (4.3)$$

$$E_{cd,max,eq} = \frac{\sigma_{cd,max,eq}}{f_{cd,fat}} \quad (4.4)$$

Where:

$R_{eq}$  is the stress ratio

$E_{cd,min,eq}$  is the minimum compressive stress level

$E_{cd,max,eq}$  is the maximum compressive stress level

$f_{cd,fat}$  is the design fatigue resistance for the concrete according to equation (4.5)

$\sigma_{cd,max,eq}$  is the maximum equivalent compressive stress for  $10^6$  cycles

$\sigma_{cd,min,eq}$  is the minimum equivalent compressive stress for  $10^6$  cycles



The design fatigue resistance of concrete according to Eurocode 2 is determined by:

$$f_{cd, fat} = k_1 \cdot \beta_{cc}(t_0) \cdot f_{cd} \cdot \left(1 - \frac{f_{ck}}{250}\right) \quad (4.5)$$

Where:

$k_1$  is a coefficient depending on the reference number of cycles till failure. According to EN 1992-1-1:2005-NA, this coefficient should be set to 1.0

$\beta_{cc}(t_0)$  is a coefficient for the concrete compressive strength at a certain age according to equation (4.6)

$t_0$  is the concrete age in days when first subjected to the fatigue loading

$f_{cd}$  is the design compressive concrete strength in [MPa]

$f_{ck}$  is the characteristic compressive concrete strength in [MPa]

The coefficient for estimation of the concrete compressive strength for a certain age is calculated according to:

$$\beta_{cc}(t_0) = e^{s \left(1 - \sqrt{\frac{28}{t_0}}\right)} \quad (4.6)$$

Where:

$s$  is a coefficient depending on the type of cement

The second method in Eurocode 2 checks the stresses in the concrete under the frequent load combination. It states that if the following condition is fulfilled the compressed concrete has adequate fatigue strength:

$$\frac{\sigma_{c, max}}{f_{cd, fat}} \leq 0,5 + 0,45 \cdot \left( \frac{\sigma_{c, min}}{f_{cd, fat}} \right) \quad (4.7)$$

$$\leq 0,9 \text{ for } f_{ck} \leq 50MPa$$

$$\leq 0,8 \text{ for } f_{ck} > 50MPa$$

Where:

$\sigma_{c,max}$  is the maximum concrete compressive stress under the frequent load combination

$\sigma_{c,min}$  is the minimum concrete compressive stress in the section where  $\sigma_{c,max}$  is found

In this project the first Eurocode concrete method will be referred to as EC2:1 and the second method as EC2:2.

## 4.2 Fatigue verification of concrete according to fib Model Code 2010

Fatigue verification of plain concrete according to fib Model Code 2010 is found in Model Code 2010, first complete draft, fib (2010). In this method the number of load cycles until fatigue failure is estimated for constant amplitude stress, in contrast to the approaches in Eurocode 2 that only considers the stresses in the concrete. As well as in Eurocode 2 a reference concrete strength for static load,  $f_{ck,fat}$ , is estimated and compared to the concrete stresses. The fatigue verification in fib Model Code 2010 can be used to check pure compression, compression-tension and pure tension failure of the concrete. Here the fatigue assessment will only include a verification of the concrete compressive strength since this can be compared to the methods in Eurocode 2.

The maximum and minimum compressive stress levels caused by fatigue loading are defined as:

$$S_{c,max} = \frac{|\sigma_{c,max}|}{f_{ck,fat}} \quad (4.8)$$

$$S_{c,min} = \frac{|\sigma_{c,min}|}{f_{ck,fat}} \quad (4.9)$$

$$\Delta S_c = |S_{c,max}| - |S_{c,min}| \quad (4.10)$$

Where:

$S_{c,max}$  is the maximum compressive stress level

$S_{c,min}$  is the minimum compressive stress level

$\Delta S_c$  is the stress range

$\sigma_{c,max}$  is the maximum compressive stress in a cycle

$\sigma_{c,min}$  is the minimum compressive stress in a cycle

For compression of the concrete the condition below should be fulfilled, and then the following equations apply:

For  $S_{c,min} > 0.8$  assume  $S_{c,min} = 0.8$

For  $0 \leq S_{c,min} \leq 0.8$

$$\log N_1 = (12 + 16S_{c,min} + 8S_{c,min}^2) \cdot (1 - S_{c,max}) \quad (4.11)$$

$$\log N_2 = 0.2 \log N_1 \cdot (\log N_1 - 1) \quad (4.12)$$

$$\log N_3 = \log N_2 \cdot (0.3 - 0.375S_{c,min}) \cdot \left( \frac{1}{\Delta S_c} \right) \quad (4.13)$$

$$\text{If } \log N_1 \leq 6 \quad (4.14)$$

then  $\log N = \log N_1$

$$\text{If } \log N_1 > 6 \text{ and } \Delta S_c \geq (0.3 - 0.375S_{c,min}) \quad (4.15)$$

then  $\log N = \log N_2$

$$\text{If } \log N_1 > 6 \text{ and } \Delta S_c < (0.3 - 0.375S_{c,min}) \quad (4.16)$$

then  $\log N = \log N_3$

Where:

$N$  is the number of load cycles until fatigue failure

The fatigue reference compressive concrete strength can be determined by:

$$f_{ck,fat} = \beta_{cc}(t_0) \cdot \beta_{c,sus}(t, t_0) \cdot f_{ck} \left( 1 - \frac{f_{ck}}{25f_{ctk0}} \right) \quad (4.17)$$

Where:

- $f_{ck}$  is the characteristic compressive strength in [MPa]
- $f_{ck,fat}$  is the reference fatigue compressive strength
- $f_{ctk0}$  is 10 MPa
- $\beta_{cc}(t_0)$  is a coefficient depending on the age of the concrete at the first application of fatigue loading
- $\beta_{c,sus}(t, t_0)$  is a coefficient that takes the effect of high mean stresses during loading into account. May be assumed to 0.85 for fatigue loading

When the number of load cycles until fatigue failure is known the Palmgren-Miner damage summation may be applied to estimate the fatigue life.

$$D = \sum_i \frac{n_{Si}}{n_{Ri}} < 1 \quad (4.18)$$

Where:

- $D$  is the fatigue damage
- $n_{Si}$  is the number of acting stress cycles at a given stress level and stress range
- $n_{Ri}$  is the number of cycles causing failure at the same stress level and stress range

### 4.3 Fatigue verification of reinforcing steel according to Eurocode 2

Fatigue verification of reinforcing steel, according to Eurocode 2, is presented in EN 1992-1-1, SIS (2005). Two different approaches for fatigue assessment of the reinforcement are given.

In the first method the Palmgren-Miner damage summation is used to estimate the damage.

$$D_{Ed} = \sum_i \frac{n(\Delta\sigma_i)}{N(\Delta\sigma_i)} < 1 \quad (4.19)$$

Where:

$n(\Delta\sigma_i)$  is the number of cycles for stress range  $\Delta\sigma_i$

$N(\Delta\sigma_i)$  is the ultimate number of cycles for stress range  $\Delta\sigma_i$

From the characteristic fatigue strength curve ( $S-N$  relation) of reinforcing and prestressing steel the ultimate number of cycles,  $N(\Delta\sigma_i)$ , can be estimated. As can be seen in Figure 4.1 the  $S-N$  relation is a curve with two different slopes. The conditions stated in equations (4.20) and (4.21) have been estimated from the  $S-N$  relation and give the ultimate number of cycles for the slope valid for a certain stress range  $\Delta\sigma_i$ .

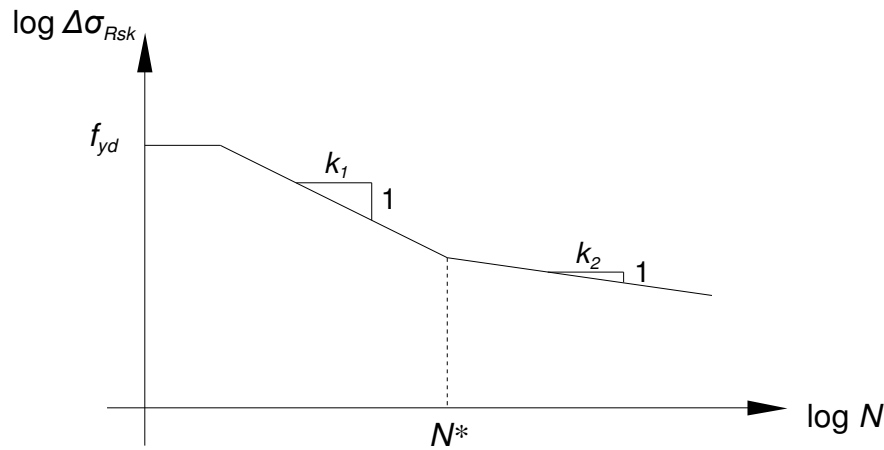


Figure 4.1  $S-N$  relations for reinforcing and prestressing steel.

$$N(\Delta\sigma_i) = N^* \cdot \left( \frac{\Delta\sigma_{Rsk} / \gamma_{S,fat}}{\gamma_{F,fat} \cdot \Delta\sigma_i} \right)^{k_1} \quad \text{if } \gamma_{F,fat} \cdot \Delta\sigma_i \geq \frac{\Delta\sigma_{Rsk}}{\gamma_{S,fat}} \quad (4.20)$$

$$N(\Delta\sigma_i) = N^* \cdot \left( \frac{\Delta\sigma_{Rsk} / \gamma_{S,fat}}{\gamma_{F,fat} \cdot \Delta\sigma_i} \right)^{k_2} \quad \text{if } \gamma_{F,fat} \cdot \Delta\sigma_i < \frac{\Delta\sigma_{Rsk}}{\gamma_{S,fat}} \quad (4.21)$$

Where:

$N^*$  is a reference value for number of cycles until fatigue failure depending on reinforcement type, Table 6.3N in EN 1992-1-1:2005

$\Delta\sigma_{Rsk}$  is the reference fatigue stress range after  $N^*$  cycles, Table 6.3N in EN 1992-1-1:2005

$\gamma_{S,fat}$	is a partial factor for fatigue that takes the material uncertainties into account, Table 2.1N in EN 1992-1-1:2005-2.4.2.4
$\gamma_{F,fat}$	is a partial factor for fatigue loading with recommended value 1.0, EN 1992-1-1:2005-2.4.2.3 (1)
$k_1$	is the exponent that defines the slope of the first part of the S-N curve, Table 6.3N in EN 1992-1-1:2005
$k_2$	is the exponent that defines the slope of the second part of the S-N curve, Table 6.3N in EN 1992-1-1:2005

The second method for fatigue verification of the reinforcement uses a damage equivalent stress range. This method can be applied on standard cases with known loads, most commonly bridges. When using it for buildings the damage equivalent stress range can be approximated with the maximum steel stress range under the existing load combination. According to this method the reinforcement has sufficient fatigue resistance if the following condition is fulfilled:

$$\gamma_{F,fat} \cdot \Delta\sigma_{S,max} \leq \frac{\Delta\sigma_{Rsk}(N^*)}{\gamma_{S,fat}} \quad (4.22)$$

Where:

$\Delta\sigma_{S,max}$  is the maximum steel stress range

$\Delta\sigma_{Rsk}(N^*)$  is the reference resisting fatigue stress range at  $N^*$  cycles

The two methods for fatigue verification of the reinforcement will in this project be referred to as EC2:1 and EC2:2.

For shear reinforcement the methods described above can be used, however the value of  $\Delta\sigma_{Rsk}$  is only valid for straight bars and therefore needs to be reduced for the bent bars in the shear reinforcement. This is done according to the following equation:

$$\xi = 0.35 + 0.026 \frac{D}{\phi} \quad (4.23)$$

Where:

$\xi$  is a reduction factor for the reference fatigue stress range for bent bars

$D$  is the bending diameter of a bent bar

$\phi$  is the diameter of the reinforcement bar

## **5 Static Design Procedure of Foundation Slabs for Wind Power Plants**

### **5.1 Properties and geometry**

When making a preliminary design of a foundation slab for a wind power plant, initial conditions need to be determined. The conditions for the location of the wind power plant are often given by the supplier of the plant with help from a geotechnical site study. The type of soil, the depth to solid ground and the groundwater level are some of these conditions. There are also requirements on the foundation slab and soil for minimum rotational stiffness around the horizontal axis and minimum stiffness for horizontal translation. These demands are set by the supplier of the plant.

For the design of the foundation slab in this project, the conditions were based on information received from Siemens for a plant located near Tuggarp, Gränna in Sweden. Some of the conditions were based on what is common for building sites of wind power plants. The conditions for the foundation slab are; friction soil, the groundwater level is below the foundation slab, the minimum demand for combined stiffness of soil and foundation is 1500MNm/deg with regard to rotational stiffness and the minimum stiffness with regard to horizontal translation is 500MN/m. The placement above the ground water level means that the pore pressure in the soil was disregarded.

The designed slab is a gravity foundation, since this is the most common type of foundation for wind power plants. The shape of the slab was assumed to be square and with a constant height, i.e. no slope at the top of the slab. Concrete class C30/37 and reinforcement type B500B was chosen. The anchor ring is placed in the centre of the slab, 1.7 meters down in the concrete. It has a height of 2.45 meters, an outer diameter of 4.2 meters and an inner diameter of 3.6 meters.

### **5.2 Loads**

Static loads acting on the slab are loads from the tower, snow and temperature actions, see Chapter 2, in particular Section 2.2. The variable loads acting directly on the foundation are small in comparison and was disregarded in this analysis. The specific wind power plant has a hub-height of 99.5 meters and the design wind speed at the height of the hub is 8.5 m/s. This has been used to find the loads acting on the foundation slab.

The design loads for the foundation slab are calculated by Siemens according to the standard for wind power plants, IEC 61400-1. This standard describes several different design load cases, for 8 design situations, that should be considered in design of members of wind power plants. From these load cases the most critical cases should be found, and the design of the member should be based on these. For each of the design load cases the appropriate type of analysis is specified, with U for analysis of ultimate loads used to ensure structural stability and material strength, or with F for load cases used in fatigue assessment of the member. The design load cases for

ultimate loads are divided into normal, abnormal and transport and erection load cases. The normal load cases are expected to occur frequently and the turbine is then in a normal state or suffering minor faults. The abnormal load cases are less likely to occur and often result in severe faults. Siemens provide information about five design load cases and state that two of these should be used for the design of the foundation slab. These two load cases are maximum design loads under normal power production and maximum design loads under extreme conditions and they are including partial safety factors from IEC 61400-1. In this standard it is stated that partial safety factors from other design codes can be used together with partial safety factors from IEC 61400-1. This is valid as long as the combined value of the partial safety factors for loads, materials and consequence of failure is not less than the combination of the partial safety factors that are stated in IEC 61400-1. In this project the design loads provided by Siemens were used including the partial safety factors from IEC 61400-1. The partial safety factors for the materials, the self weight of the slab and the horizontal soil pressure were according to Eurocode 2. It was ensured that the combined value of the safety factors was at least what is stated in IEC 61400-1. Table 5.1 shows the partial safety factors according to IEC 61400-1 and Eurocode 2.

*Table 5.1 Partial safety factors according to IEC 61400-1 and partial safety factors in the ultimate limit state according to Eurocode 2.*

		IEC 61400-1		Eurocode 2	
Loads		Normal	$\gamma_f = 1,0$ or 1,35	Permanent	$\gamma_G = 1,0$ or 1,35
		Abnormal	$\gamma_f = 1,1$	Variable	$\gamma_Q = 1,50$
		Transport	$\gamma_f = 1,5$		
Material		$\gamma_m \geq 1,1$		Concrete	$\gamma_C = 1,50$
				Steel	$\gamma_S = 1,15$
Consequence of failure	Class 1	$\gamma_n = 0,9$		0,9	
	Class 2	$\gamma_n = 1,0$		1,0	
	Class 3	$\gamma_n = 1,3$		1,1	

The foundation design loads provided by Siemens are given as sectional forces for the normal load case based on normal operation and the abnormal load case which is the load case with the highest overturning moment. From the tower to the foundation slab the following sectional forces due to loading are transferred; normal force, horizontal force, overturning and torsion moment. The foundation slab design in the ultimate limit state was based on the design load case which resulted in the highest sectional



forces and this is the one with the highest overturning moment, see Table 5.2. For design of the slab in serviceability limit state the design load case for normal operation was used.

*Table 5.2 Foundation design loads, including partial safety factors according to IEC 61400-1, for normal operation and design load case with highest overturning moment.*

	Normal operation	Highest overturning moment
Normal force, $F_z$	3 600 kN	3 600 kN
Horizontal force, $F_{res}$	800 kN	1 080 kN
Overturning moment, $M_{res}$	72 500 kNm	97 700 kNm
Torsion moment, $M_z$	7 900 kNm	3 800 kNm

### 5.3 Design of the foundation slab in the ultimate limit state

For the static design of the foundation slab there are several aspects that need to be considered. The stability of the slab has to be verified for resistance against overturning moment and required rotational stiffness. The slab must also have a sufficient flexural resistance with regard to positive and negative bending moments. Further the shear capacity of the slab must be adequate, and if necessary shear reinforcement has to be designed.

For a foundation slab for a wind power plant the anchor ring transfers forces from the tower to the concrete slab by embedded anchor bolts. The lower part of the anchor ring, that is embedded in the concrete slab, provides the anchorage of the tensile forces from the tower. The connection with the concrete is done by geometrical locking at the lower part of the anchor ring. The choice with embedded bolts enables post-tensioning of the interface between the tower and the foundation slab and this is preferable since it is increasing the fatigue life of the structure. The compressive forces from the tower can be assumed to be transferred directly from the upper part of the anchor ring, which is not embedded in the concrete slab, to the top of the foundation slab, see Figure 5.1. Due to these compressive forces punching shear needs to be checked under the loaded area of the ring.

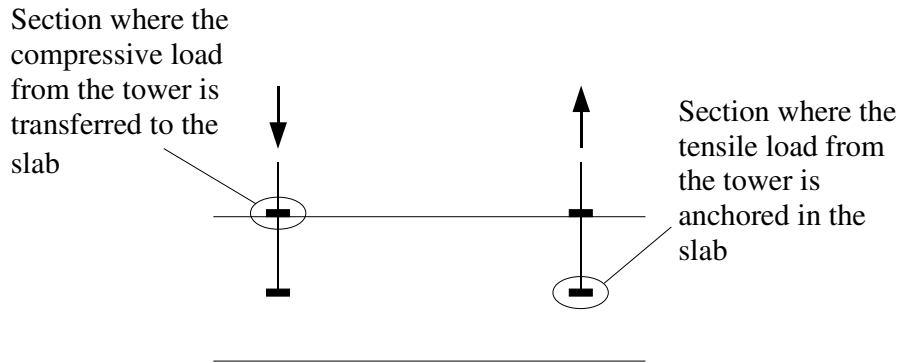


Figure 5.1 Sections where the compressive and tensile loads from the tower are transferred.

Compression of the concrete under the loaded area of the anchor ring also needs to be checked since it can be considered as a concentrated compression node in a strut and tie model. All of these verifications were performed according to Eurocode 2.

### 5.3.1 Stability and sizing

The demands on stability of the foundation slab will determine the size of it. In order to check stability, the slab size was assumed and the reaction force from the ground was checked to be acting under the slab. If the reaction force is acting outside the slab this would imply that the stability against overturning moment is not sufficient. The location of the reaction force,  $R_{res}$ , was found by moment equilibrium for the self weight of the wind power plant and the slab, and the sectional forces as well as the horizontal soil pressure, see Figure 5.2. This resulted in an estimation of  $b$ , which is the distance from the edge of the foundation slab to the gravity centre of the reaction force. The reaction force is uniformly distributed over the length  $2b$ . The compressed area under the slab was then found to verify that the ground could resist the soil pressure caused by the slab.

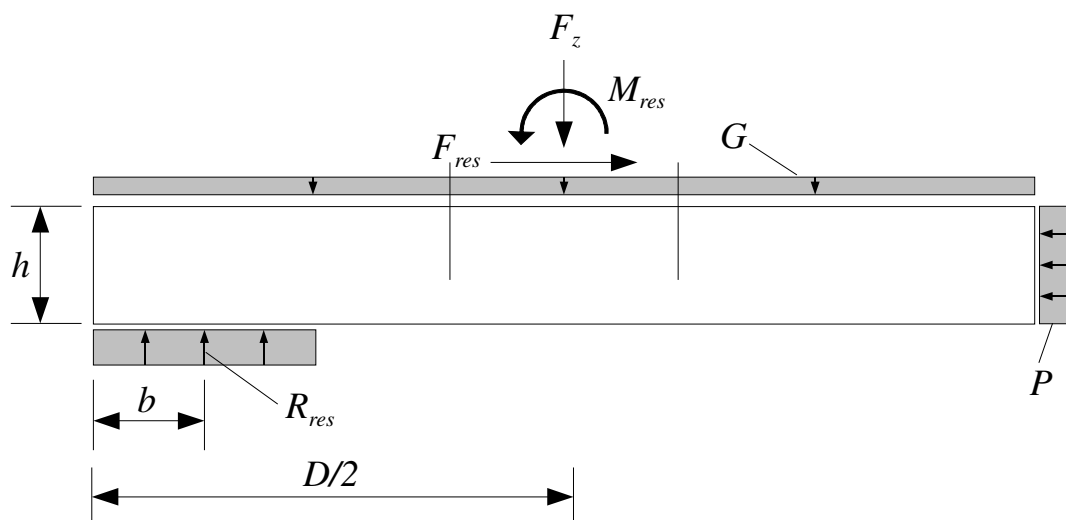


Figure 5.2 Model of the loads acting on the slab and the reaction force from the ground.

To check the combined rotational stiffness of the slab and soil the slab was assumed to act as a cantilever and the rotation caused by the sectional moment was calculated. The reaction force from the ground is a uniformly distributed load over the length  $2b$  and the self weight of the slab was assumed to counteract the rotation.

### 5.3.2 Force distribution

The force distribution in the slab was estimated by using a load model where the overturning moment, the horizontal force and the normal force were modelled as a force couple, see Figure 5.3. This force couple consists of a compressive force,  $F_C$ , and a tensile force,  $F_T$ . These forces are transferred to the foundation slab through the anchor ring and the reaction force from the ground,  $R_{res}$ , is counteracting these forces and the self weight of the slab.

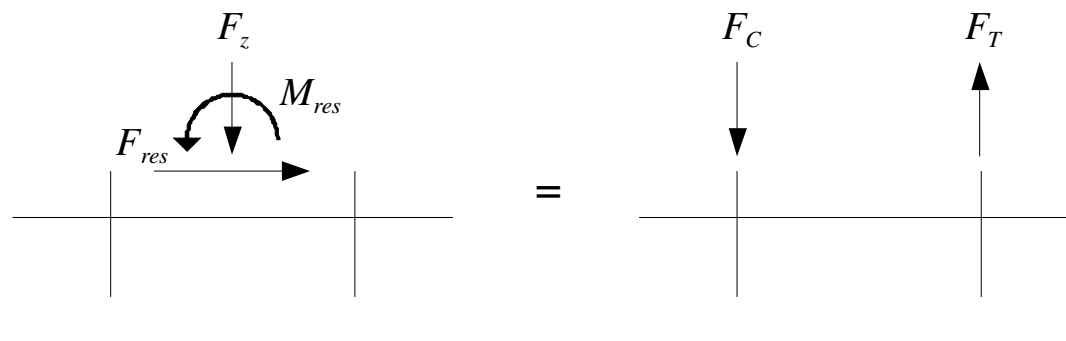


Figure 5.3 The loads that are acting on the slab and how they are modelled as a force couple  $F_C$  and  $F_T$ .

Compression of the concrete is concentrated to a small part of the slab under the loaded area of the anchor ring. By using the model shown in Figure 5.4, assuming a simplified stress block approach, the height of the compressed part of the anchor ring was obtained. When this height was known the distance between the compressive and tensile force in the force couple could be found.

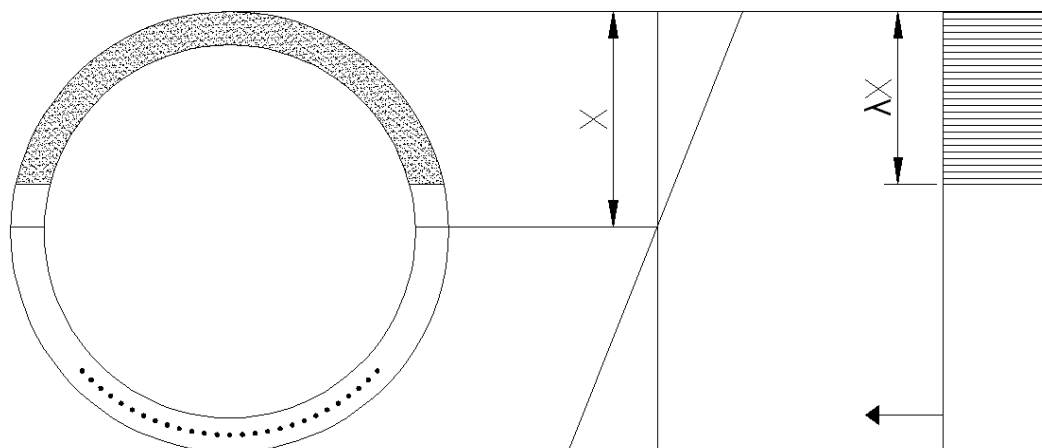


Figure 5.4 Model to find the height of the compressed part of the anchor ring.

In order to determine the height of the compressive zone the part of the anchor ring that is in tension had to be assumed. In this case a quarter of the ring was assumed to be in tension, which means that the anchor bolts in this quarter are the ones that are active when it comes to transfer of tensile forces to the slab. The average depth to the anchor bolts in the quarter of the ring was estimated. With these assumptions the height of the compressive zone could be calculated and then the distance between the compressive and the tensile force could be found.

A moment diagram was obtained from the force distribution, see Figure 5.5. In the diagram it can be seen that in this case the critical sections for the bending moment were found under the force couple. The maximum bending moment found under  $F_C$  was used to estimate the amount of bottom reinforcement needed. The bending moment under the force  $F_T$  was used to estimate the needed amount of top reinforcement.

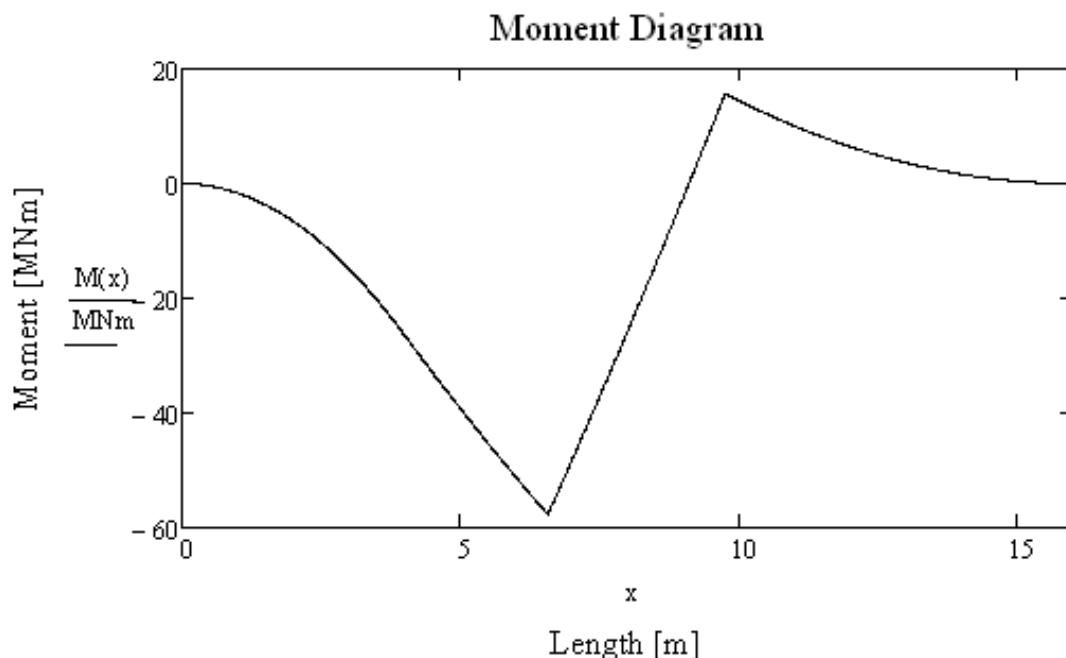


Figure 5.5 Assumed moment distribution along the slab.

The shear capacity of the foundation slab was verified in transverse sections across the slab assuming the slab to behave like a beam. See Figure 5.6 for the shear force distribution along the slab. The maximum shear force was found between the force couple, i.e. inside the anchor ring. The slope at the left part of the diagram is due to the distributed reaction force,  $R_{res}$ .

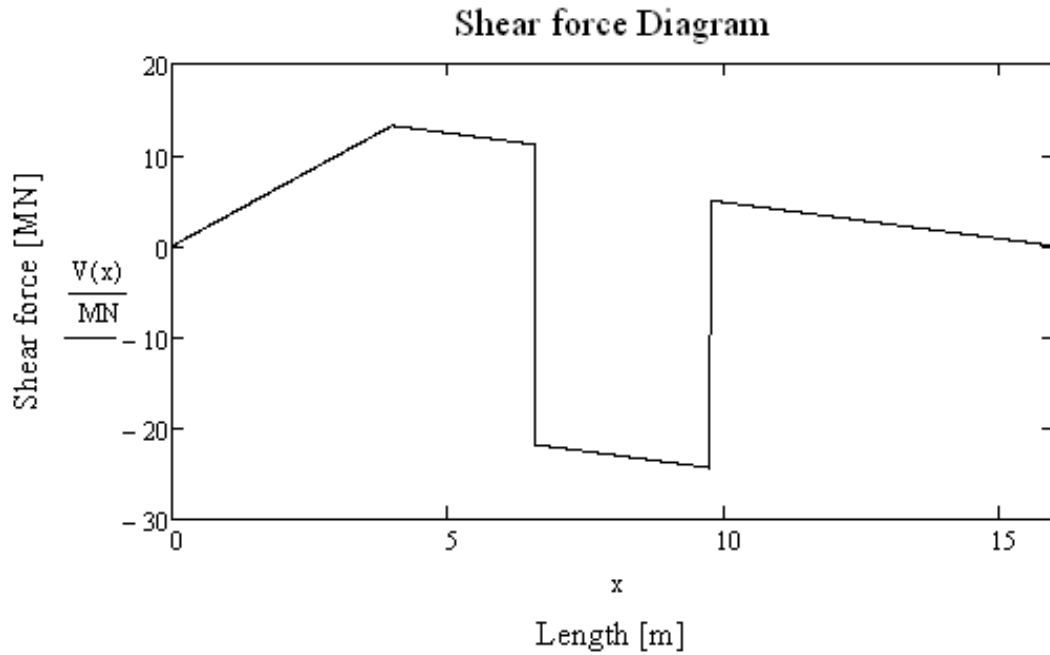


Figure 5.6 Shear force distribution along the slab.

### 5.3.3 Strut and tie model and design of compression-compression node

A strut and tie model was established to find the required reinforcement amounts and the horizontal compressive force acting on the compressed concrete area under the anchor ring. In the strut and tie model the load model with a force couple was used, see forces  $F_C$  and  $F_T$  in Figure 5.7. The gravity centre of the reaction force from the ground is on a distance  $b$  from the edge of the slab, as estimated previously. This reaction force is a uniformly distributed load but in the strut and tie model it was modelled as a concentrated force acting on the distance  $b$ . The self weight of the slab was divided into two force components that are acting in nodes outside the force couple. The forces in each strut and tie were calculated and the stresses in the compression-compression node, node 5 in Figure 5.7, were calculated to design this node for compression failure.

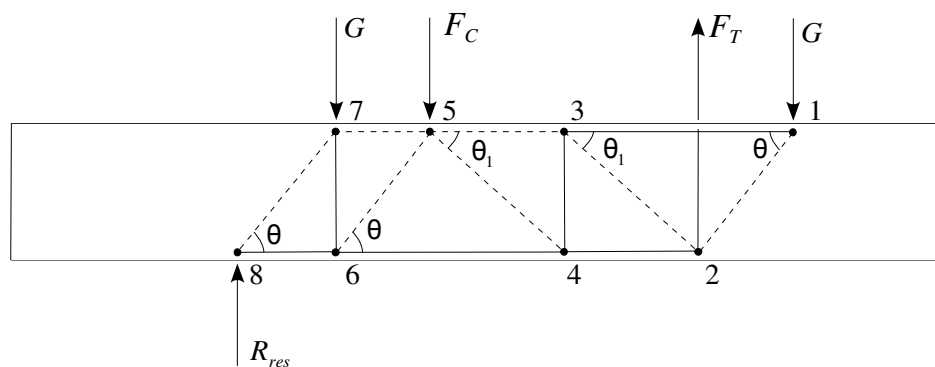


Figure 5.7 The strut and tie model used in the preliminary design of the slab.

The capacity of the concrete in the compression-compression node under the anchor ring is increased due to embedded concrete:

$$\sigma_{Rd,max} = k_4 \cdot \nu \cdot f_{cd} \quad (5.1)$$

Where:

$$\nu = 1 - \frac{f_{ck}}{250} \quad (5.2)$$

Where:

$\sigma_{Rd,max}$  is the largest stress that can be applied at the edge of the node, EN 1992-1-1:2005-6.5.4 (4)

$k_4$  is a parameter influencing the design strength for concrete struts with recommended value 3.0, EN 1992-1-1:2005-6.5.4 (4)

$\nu$  is a parameter influencing the design strength for concrete struts, EN 1992-1-1:2005-6.5.2 (2)

$f_{cd}$  is the design compressive concrete strength in [MPa]

$f_{ck}$  is the characteristic compressive strength in [MPa]

### 5.3.4 Flexural resistance

From the strut and tie model the tensile forces in the different parts of the foundation slab were known. There are tensile forces in a part of the top of the slab as well as along the entire bottom of it. Two vertical ties are shown in the model, see Figure 5.7 above, and these indicate that there is a need for shear reinforcement, both inside and outside the anchor ring. The amount of reinforcement was estimated with regard to resistance of the tensile forces in the slab, see equation (5.3).

$$A_s = \frac{T}{f_{yd}} \quad (5.3)$$

Where:

$A_s$  is the required reinforcement amount

$T$  is the tensile force estimated in the strut and tie model

$f_{yd}$  is the design yield strength of the reinforcement

### 5.3.5 Shear capacity

The slab was checked for sufficient shear capacity by assuming the critical shear crack at the edge of the soil reaction. It was also necessary to verify the shear capacity

in punching shear, due to the transfer of forces through the anchor ring. By assuming that the tower acts like a circular column support, the calculation was carried out according to Eurocode 2, EN 1992-1-1:2005 6.4. The anchor ring is assumed to transfer the compressive forces from the tower to the slab at the surface of the slab. The depth on which punching shear should be checked was therefore the whole depth of the slab. In the calculations for punching shear capacity different critical perimeters were assumed. The highest inclination checked was 40 deg and the lowest 26 deg, see Figure 5.8. If the shear capacity of the concrete slab is insufficient shear reinforcement is needed and if this is the case the shear reinforcement is assumed to carry the whole shear force.

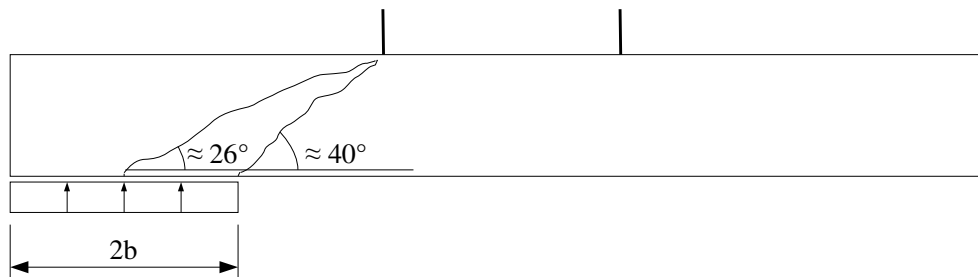


Figure 5.8 Crack inclination for the different critical perimeters used in calculation of punching shear.

## 5.4 Remarks and conclusions from the static design

The maximum and the minimum amount of reinforcement were verified in both ultimate- and serviceability limit state. The crack width in serviceability limit state was compared to the maximum allowed crack width, which is dependent on the life span of the slab and the exposure class. For this slab the life span class was set to L50 and the exposure class to XC2, this gave a maximum allowed crack width of 0.45 mm.

To meet the demands for stability of the slab the dimensions were chosen to 16x16 meters and the height was set to 2 meters. The reinforcement arrangement was chosen to consist of two layers in the bottom of the slab, to ensure sufficient flexural resistance. The bars are 24 mm in diameter and the spacing is 150 mm. The top reinforcement was also placed in two layers with bars with the same diameter as the bottom reinforcement. The dimensions of the slab as well as soil pressure, rotational stiffness and reinforcement amounts are shown in Table 5.2.

Table 5.2 Preliminary design of the foundation slab, showing width, height, soil pressure, rotational stiffness, bottom, top and shear reinforcement.

Width [m]	Height [m]	Soil pressure [kPa]	Rotational stiffness [MNm/deg]	$A_s$ [mm <sup>2</sup> ]	$A'_s$ [mm <sup>2</sup> ]	$A_{sv}$ [mm <sup>2</sup> ]
16	2	237,0	17 877	76 812	18 621	54 329

In the strut and tie model the force  $F_T$  is applied at the top of the slab and transferred down to node 2 at the level of the bottom reinforcement, see Figure 5.7 above. The force  $F_T$  is however transferred into the slab via the anchor ring and the bottom of the anchor ring is in reality not at the level of the bottom reinforcement. In order to model the behaviour of the slab accurately with the strut and tie model, a connection between the bottom reinforcement and the lower part of the anchor ring was assumed.

The compression node under the force  $F_C$  was assumed to be a critical section with regard to crushing of the concrete. In the initial static calculations the compressive capacity in this node was not sufficient. No limitations on the height of the node could be found, so to increase the capacity of the node the height was increased and sufficient capacity obtained. However, with an increase of node height the angles in the strut and tie model change and a redistribution of forces will occur.



## 6 Fatigue Verification of Foundation Slab for Wind Power Plant

### 6.1 Fatigue load calculations

The fatigue loads are provided by Siemens and are calculated for a 20 year operating time. They are shown as spectrum in three different tables: one for horizontal force, one for overturning moment and the last one for torsion. Each of the fatigue loads obtained in the table will have a damaging impact on the foundation slab. The total fatigue damage of all the loads will therefore be the damage caused over the whole period of 20 years. In order to estimate this fatigue damage, the moments affecting the fatigue calculation has to be found.

The values in the table are the fatigue loads that the wind gives rise to. These fatigue loads come with the number of cycles shown on the left axis, see Table 6.1. The numbers of cycles seen in the table is the sum of the cycles for all the loads in that row.

Table 6.1 *Fatigue load spectrum for overturning moment.*

Overturning moment. Peak-to-peak fatigue load bins [kNm]											
		Mean value [kNm]									
		-26 200 to -16 400	-16 400 to -6 500	-6 500 to 3 300	3 300 to 13 200	13 200 to 23 000	23 000 to 32 900	32 900 to 42 700	42 700 to 52 600	52 600 to 62 400	62 400 to 72 200
Number of cycles	1.00E+09	0	0	0	0	0	0	0	0	0	0
	5.00E+08	0	0	0	0	0	0	0	0	0	0
	2.00E+08	0	0	0	10	10	10	0	0	0	0
	1.00E+08	0	0	0	10	2 000	2 000	10	0	0	0
	5.00E+07	0	0	10	2 000	2 000	4 000	10	0	0	0
	2.00E+07	0	0	10	2 000	6 000	6 000	2 000	0	0	0
	1.00E+07	0	0	10	2 000	9 900	9 900	4 000	0	0	0
	5.00E+06	0	0	2 000	4 000	13 900	13 900	6 000	0	0	0
	2.00E+06	0	0	2 000	4 000	15 900	17 800	8 000	0	0	0
	1.00E+06	0	0	2 000	6 000	19 800	21 800	9 900	0	0	0
	5.00E+05	0	0	4 000	9 900	21 800	25 800	11 900	0	0	0
	2.00E+05	0	0	6 000	13 900	25 800	27 700	11 900	0	0	0
	1.00E+05	0	0	8 000	15 900	27 700	29 700	13 900	0	0	0
	5.00E+04	0	0	9 900	17 800	13 700	31 700	13 900	10	0	0
	2.00E+04	0	0	13 900	19 800	35 600	35 600	15 900	10	0	0
	1.00E+04	0	0	13 900	19 800	39 600	35 600	17 800	10	0	0
	5.00E+03	0	0	19 800	21 800	47 500	39 600	25 800	10	0	0
	2.00E+03	0	10	23 800	31 700	57 400	39 600	25 800	2 000	0	0
	1.00E+03	2 000	41 600	59 400	87 100	89 000	98 900	79 200	8 000	2 000	2 000

At the top of the table the mean values for the loads are shown. These mean values are given by a range and the mean value can be chosen as any value within this range. In the fatigue assessment performed in this project the mean value was chosen as the mean value of the specific range.

There are 93 loads in the spectrum shown for overturning moment. For each of these loads a mean value was estimated to simplify the calculations. If for example bin number 5 is used, the mean value should be between 13200 and 23000 kNm, which is the range for the bin. In this project the mean value was chosen to the mean of the range for the bin, i.e. 18100 kNm. The value in the table is then the peak to peak value, i.e. the difference between the minimum and the maximum value in a cycle, which is the same as the amplitude,  $M_{fat.amplitude}$ , see Figure 6.1. The maximum

moment is the mean value plus half the amplitude,  $\Delta M$ , and the minimum value is the opposite, the mean value minus half the amplitude.

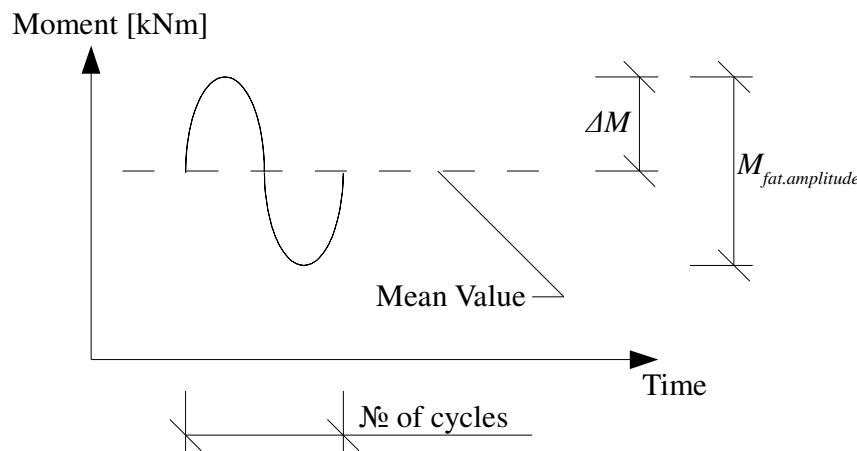


Figure 6.1 The fatigue load for one amplitude in the overturning moment table.

The moment that affects the fatigue calculations is the difference between the maximum and the minimum moment if both of these are positive. If the minimum value is negative the fatigue moment used in the calculations is equal to the maximum moment since a negative value implies that the slab is subjected to compression instead of tension. If both the maximum and the minimum values are negative the fatigue moment is set to zero.

## 6.2 Strut and tie models to find fatigue stresses

For the fatigue assessment the stresses due to fatigue loading had to be checked. The strut and tie model used in the static design was assessed to see if this could be used for serviceability limit state, and describe the behaviour of the slab when it is subjected to fatigue. Due to the large variations in the fatigue moments the strut and tie model was however found not to work properly for all the loads. The changes of the model for the different loads are depending on the position of the reaction force from the ground. This reaction force is acting further towards the edge of the slab for increasing fatigue moments. In the static design the reaction force was estimated to be acting on a distance,  $b$ , from the edge of the slab, and was modelled as a concentrated force. For small fatigue moments the reaction force is acting inside the anchor ring and this result in a different strut and tie model. For the larger moments where the reaction force is acting closer to the distance  $b$ , that was estimated in the static design, a strut and tie model similar to the static one can be used. When the reaction force is acting between the anchor ring and the distance used in the static design, the angles in the model will determine the number of nodes and ties. The fatigue stresses were found by establishing the strut and tie models for fatigue assessment. Because of the varying fatigue loading four different strut and tie models were established to describe the stress field properly. In all four models the distance between  $F_T$  and  $F_C$  was assumed to be the average diameter of the anchor ring instead of the distance

between the compressive and tensile part of the ring, which was the distance used in the static design.

The first two strut and tie models are both simulating the stress field in the slab when it is subjected to small fatigue moments, which results in that the reaction force from the ground is acting inside the anchor ring. The fatigue moments result in that compressive forces as well as small tensile forces are transferred to the slab. In order to get the models to function as precisely as possible the reaction force from the ground was divided into two force components. As a simplification when dividing the reaction force in two components, they were assumed to be of equal size on the same distance from the gravity centre of the reaction force.

The first strut and tie model is shown in Figure 6.3. In this model the component of the reaction force acting close to the tensile force  $F_T$  was locked under the force in node 2, while the reaction force and the component on the other side changes for the different fatigue moments. This was to simplify the calculations regarding the reaction force from the ground. This strut and tie model is valid until the distance between the locked component of the reaction force, and the actual reaction force is too large. In order to ensure a proper result this model was used until the component on the left side of the reaction force and the node it is acting in, which is node 6, was under node 7.

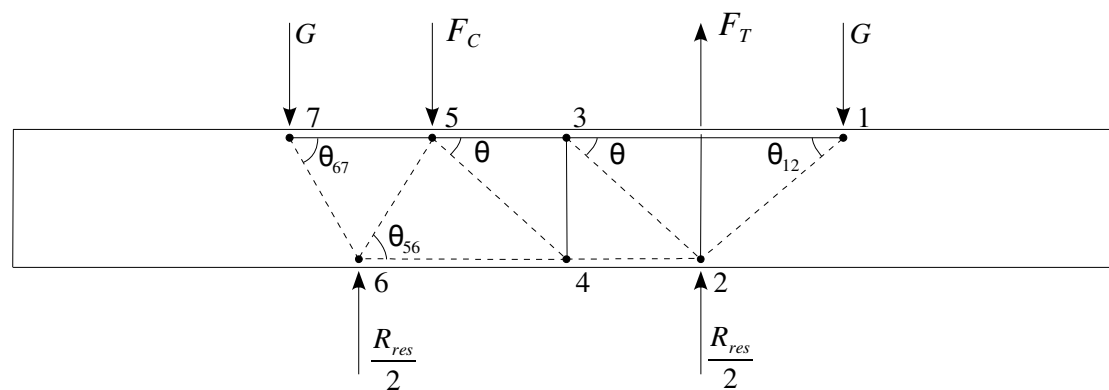


Figure 6.3 The strut and tie model for when the reaction force is acting inside the anchor ring and one of the components is locked under the tensile force.

The second strut and tie model, see Figure 6.4, was also used when the reaction force is acting inside the anchor ring. In this model the component of the reaction force acting close to the tensile force,  $F_T$ , was locked in the middle of the slab in node 4. This model is valid until the reaction force is acting under node 5, i.e. until the reaction force is no longer acting inside the ring.

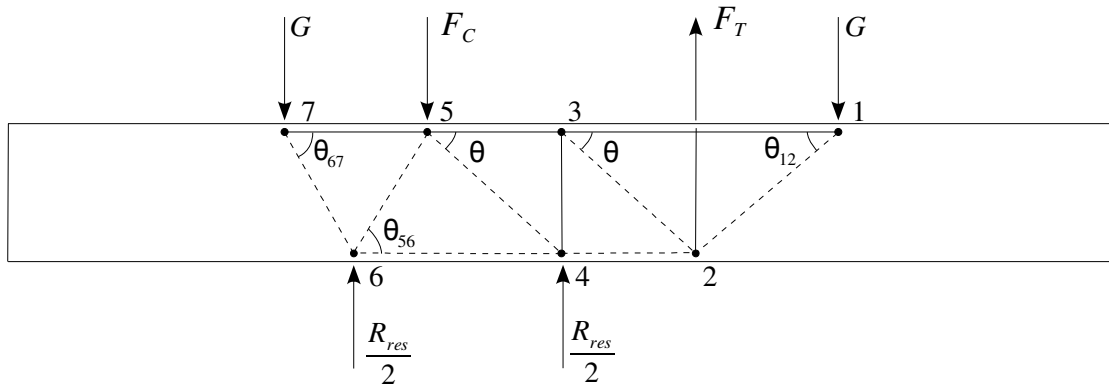


Figure 6.4 The strut and tie model for when the reaction force is acting inside the anchor ring and one of the components is locked in the middle of the slab.

The third strut and tie model, see Figure 6.5, was used to simulate the stress field in the slab when it is subjected to greater fatigue moments. This will result in that the reaction force from the ground is acting outside the anchor ring. The reaction force was modelled as a concentrated force acting on the distance  $b$  from the edge of the slab, since this gives a proper strut and tie model for this case. This model is closer to the strut and tie model used in the static design. However, the distance  $b$  that the reaction force is acting on is larger, and closer to the anchor ring, than the one estimated in the static design. The model simulates the stress field in the slab properly until the reaction force is acting under node 7.

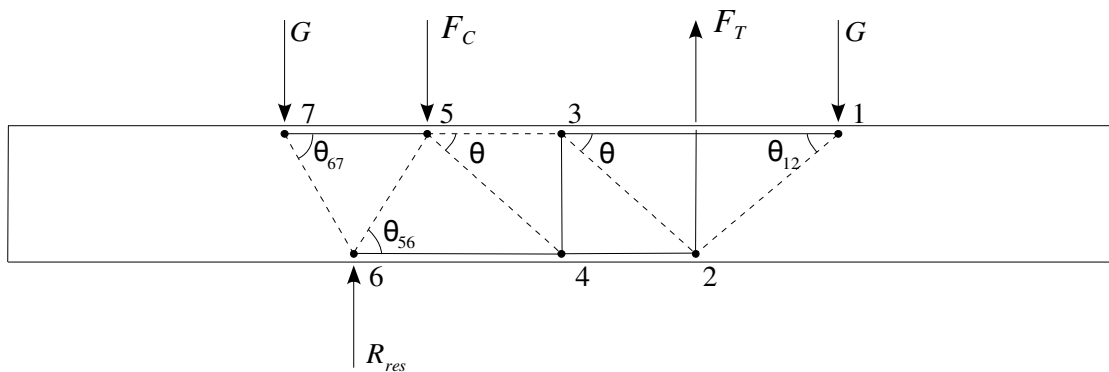


Figure 6.5 The strut and tie model for when the reaction force from the ground is acting outside the anchor ring, but not further away than node 7.

The fourth strut and tie model simulates the stress field in the slab when the reaction force from the ground is acting outside the anchor ring, on a distance further away than node 7, see Figure 6.6. The model is valid when the fatigue moments are great and result in that large tensile forces as well as large compressive forces are transferred to the slab. This model is the one closest to the strut and tie model used in the static design and is used for the smallest values of  $b$ . The reaction force was in this model, as well as in the static one, modelled as a concentrated force acting under the slab. As can be seen in the figure below the differences from the third strut and tie model is the additional vertical tie and node 8 was added.

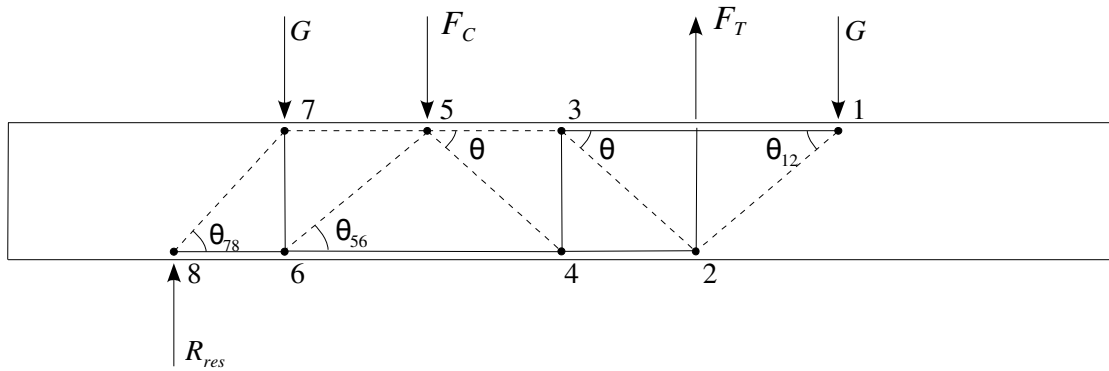


Figure 6.6 The strut and tie model for when the reaction force from the ground is acting outside the anchor ring and further away than node 7.

### 6.3 Critical sections for fatigue assessment

For fatigue verification of the foundation slab a number of sections in the slab can be assessed. For the verification in this project these sections needed to be limited to a reasonable amount. This was done by finding the critical sections and assessing these. The static analysis of the slab showed what sections were critical in the ultimate limit state, and these are then probably also critical with regard to fatigue. In order to check fatigue of the concrete as well as the reinforcement the critical sections for each of these needed to be found. This resulted in that some sections were assessed only with regard to fatigue of the reinforcement and some only with regard to fatigue of the concrete, since the critical sections for these do not coincide.

As can be seen in the static analysis, for a foundation slab of a wind power plant compression of the concrete under the tower is a critical point. This section was therefore assessed with regard to fatigue. The bottom reinforcement close to the anchor ring is subjected to fatigue stresses and needed to be verified. The section of interest for the top reinforcement was chosen to the tie between the force couple. This is because this is the only section of the top reinforcement where there are any fatigue stresses. In the tie between outside and inside the anchor ring the only force acting is the self weight of the slab and no fatigue stresses were obtained.

For the load model where the moment is modelled into a force couple, there are high compressive stresses that need transfer through the inside of the anchor ring. This stress affects the shear reinforcement and this needed to be checked for fatigue effects. The shear reinforcement outside the anchor ring was found to not be affected by the fatigue load, only the self weight.

Strut and tie models 1 and 2 were used to assess fatigue of the top reinforcement, the concrete in the compression node under the compressive force and the shear reinforcement inside the anchor ring. These sections are shown in Figure 6.7.

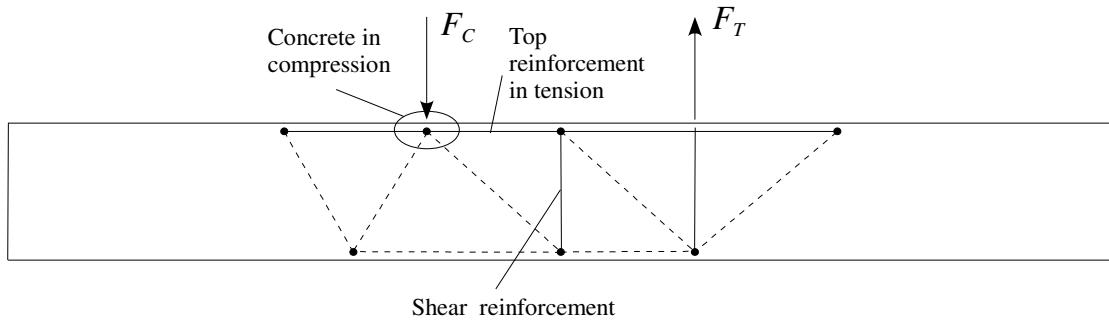


Figure 6.7 The critical sections in strut and tie models 1 and 2 that were assessed with regard to fatigue.

The sections assessed with strut and tie model 3 were the bottom and top reinforcement, the shear reinforcement inside the anchor ring and the concrete in the compression node under the compressive force, see Figure 6.8.

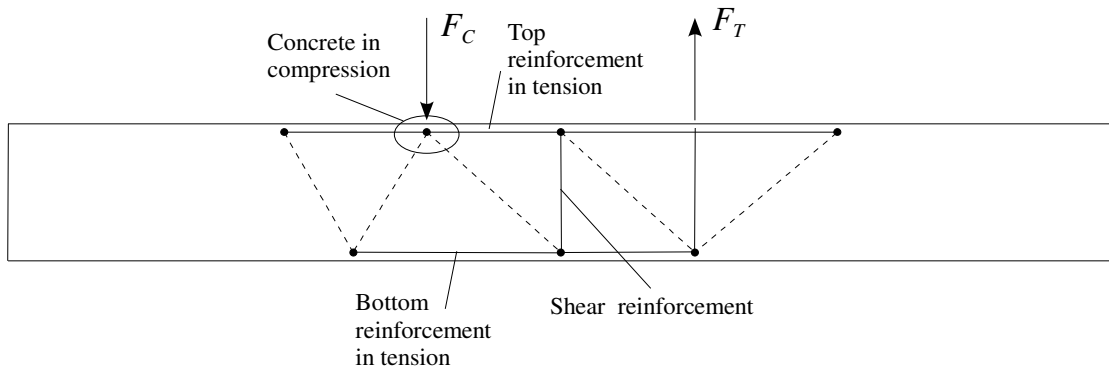


Figure 6.8 The critical sections in strut and tie model 3 that were assessed with regard to fatigue.

Strut and tie model 4 was used to assess the bottom reinforcement, the shear reinforcement inside the anchor ring and the concrete in the compression node. In Figure 6.9 the sections assessed with this strut and tie model are shown.

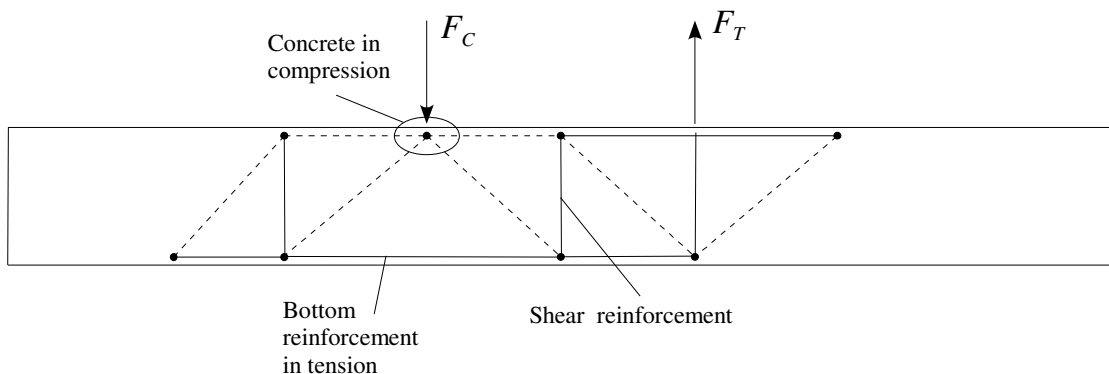


Figure 6.9 The critical sections in strut and tie model 4 that were assessed with regard to fatigue.

### 6.3.1 Fatigue stresses in the critical sections

In order to perform a fatigue assessment of the designed concrete foundation slab the sectional stresses needed to be determined. The stresses in the slab are depending on if the section is cracked or not. For a cracked reinforced concrete member the reinforcement is assumed to carry all the tensile forces. If the member would be uncracked this would imply that no tensile forces are carried by the reinforcement, while there would be no fatigue of this. In all fatigue calculations however, the cross-section should be assumed to be cracked and the tensile strength of the concrete should be disregarded in the assessment.

The stresses in the critical sections, which were chosen for the fatigue assessment, were found with the use of the results from the strut and tie models. The forces in all struts and ties were calculated for the minimum and maximum value in every cycle of fatigue loading. The stresses were calculated for both the minimum and maximum values and the tensile stresses in the different types of reinforcement were estimated by equations (6.1) and (6.2).

$$\sigma_{s,\max} = \frac{T_{\max}}{A_s} \quad (6.1)$$

$$\sigma_{s,\min} = \frac{T_{\min}}{A_s} \quad (6.2)$$

Where:

$\sigma_{s,\max}$  is the maximum tensile steel stress in a cycle

$\sigma_{s,\min}$  is the minimum tensile steel stress in a cycle

$T_{\max}$  is the maximum tensile force in the reinforcement in a cycle

$T_{\min}$  is the minimum tensile force in the reinforcement in a cycle

$A_s$  is the reinforcement amount

The fatigue stresses in the concrete were only checked in the compression-compression node under the anchor ring, since this is the most critical section for the concrete. These stresses were estimated in the same way as in the static design, by finding the area that the horizontal compressive force is acting on. This stress is the highest stress in the node and it was therefore the one checked for the fatigue assessment of the concrete.

$$\sigma_{c0,\max} = \frac{C_{\max}}{A_{c0}} \quad (6.3)$$

$$\sigma_{c0,min} = \frac{C_{min}}{A_{c0}} \quad (6.4)$$

Where:

$\sigma_{c0,max}$  is the maximum compressive concrete stress in a cycle

$\sigma_{c0,min}$  is the minimum compressive concrete stress in a cycle

$C_{max}$  is the maximum compressive concrete force in a cycle

$C_{min}$  is the minimum compressive concrete force in a cycle

$A_{c0}$  is the area the horizontal compressive force is acting on

When performing the fatigue assessment the concrete strength was increased in the same way as in the static design. This is due to the fact that the compression-compression node that was checked is embedded in the concrete.

## 6.4 Fatigue verification of foundation slab

The designed foundation slab was assessed to ensure a fatigue life of at least 20 years, since this is the estimated life span for the wind power plant. A first check of the stresses affecting the fatigue life of the slab, showed that the horizontal force and torsion moment are small in comparison to the overturning moment and have no or very small impact on the fatigue life. The fatigue loads due to horizontal force and torsion were therefore disregarded in the fatigue verification of the foundation slab.

The preliminarily designed slab has a concrete strength class of C30/37 and reinforcement amounts according to Table 6.1.

*Table 6.1 Reinforcement amounts estimated in the preliminary design.*

Bottom reinforcement [mm <sup>2</sup> ]	Top reinforcement [mm <sup>2</sup> ]	Shear reinforcement [mm <sup>2</sup> ]
76 812	18 621	54 329

In the fatigue assessment presented below the following abbreviations will be used; Eurocode 2 will be referred to as EC2 and fib Model Code 2010 as MC2010.



### 6.4.1 Fatigue verification regarding compression of the concrete

Fatigue verification of the concrete in the compression-compression node under the anchor ring was performed according to methods in EC2 and MC2010. The methods in EC2 uses, as mentioned previously, a reference value of number of cycles and do not take the frequency of the fatigue load into account. For a spectrum load, like the wind acting on a wind power plant, these methods were found to not be viable. In order to use the first of these methods an equivalent stress at the reference number of cycles has to be found. For a proper use of the second method a mean value of the maximum and minimum stresses has to be estimated, with the assumption that this is the stress under the frequent load combination. This implies that simplifications of the spectrum load are needed, such as finding a mean value for the fatigue stresses for fatigue loads with number of cycles close to the reference value. This was though not the choice for the fatigue assessment in this project. Instead the expression for the first method for fatigue verification of concrete was re-written according to the method in EN 1992-2:2005 that is commonly used for bridges, see equation (6.5). When the number of cycles until fatigue failure was known, the Palmgren-Miner rule was used to estimate the damage.

$$N = 10^{14 \cdot \frac{1 - E_{cd, \max_i}}{\sqrt{1 - R_{eq_i}}}} \quad (6.5)$$

Where:

$N$  is the number of load cycles until fatigue failure

$E_{cd, \max_i}$  is the maximum compressive stress level

$R_{eq_i}$  is the stress ratio

The method from MC2010 also estimates the number of cycles until fatigue failure, for each of the given fatigue moments. This approach was found to work well for the spectrum load since it takes the varying frequency of the different loads into account, by the use of the Palmgren-Miner summation.

The fatigue assessment regarding compression of the concrete, showed that there is no substantial damage of the concrete under the anchor ring, see Table 6.2. This is valid for all the methods used in the fatigue verification of the concrete. The MC2010 method resulted in no damage at all, while the EC2:1 method showed a damage of  $3.77 \cdot 10^{-6}$ . This damage is however insignificant. The second EC2 method also indicated that the compressed concrete had sufficient fatigue life.

*Table 6.2 Damage of the concrete in compression according to EC2:1 and MC2010, and the result for EC2:2.*

	MC2010 Damage [ - ]	EC2:1 Damage [ - ]	EC2:2
Concrete	0	$3,77 \cdot 10^{-6}$	OK

## 6.4.2 Fatigue verification regarding tensile reinforcement

The fatigue assessment of the reinforcement was performed according to two methods from Eurocode 2. One of these methods estimates the number of cycles until fatigue failure and the other uses a reference value for the number of cycles. The first method was found to work well for the spectrum load for the same reasons as the concrete method according to MC2010; it takes the frequency of the fatigue load into account. The second method is based on the same approach as the concrete methods in EC2 and is therefore found to not work properly for the spectrum load, as can be seen in the results.

The fatigue assessment regarding tensile reinforcement showed that the damage of the top reinforcement was high, while this amount had to be increased due to fatigue, see Table 6.3. According to the results from EC2:2 both top and bottom reinforcement amounts would need to be increased. However, the damage of the bottom reinforcement was low and this was left without further consideration.

*Table 6.3 Damage of top and bottom reinforcement according to EC2:1 and results for EC2:2.*

	EC2:1 Damage [ - ]	EC2:2
Top reinforcement	62,155	NOT OK
Bottom reinforcement	0,00254	NOT OK

To ensure that the slab had a 20 year fatigue life the top reinforcement amount was increased. When increasing the amount of reinforcement the damage is reduced, see Table 6.4. The amount was increased until the damage was below one, i.e. until the fatigue life of the top reinforcement was sufficient. This resulted in a final fatigue design, according to EC2:1, with an amount of top reinforcement of 38 500 mm<sup>2</sup>. For a proper fatigue design according to EC2:2 the amount of top reinforcement needed to be further increased to 51 900 mm<sup>2</sup>.

*Table 6.4 Increase of top reinforcement amount for fatigue design, according to methods in EC2.*

	Static design	Fatigue design: EC2:1	Fatigue design: EC2:2
Reinforcement amount [mm <sup>2</sup> ]	18 621	38 500	51 900
EC2:1 Damage [ - ]	62,155	0,997	0,086
EC2:2	NOT OK	NOT OK	OK

### 6.4.3 Fatigue verification regarding shear

To check the fatigue resistance of the shear reinforcement, the same methods as for the top and bottom reinforcement were used, with a reduction of the fatigue stress range because of bent bars, see Chapter 4 in particular Section 4.3 and equation (4.21). Table 6.5 shows the result from the fatigue verification of the shear reinforcement. As can be seen the fatigue assessment of the shear reinforcement indicated damage above one, which means that the fatigue life is insufficient. The result for the second method also showed that the amount of shear reinforcement would need to be increased due to fatigue.

*Table 6.5 Damage of shear reinforcement according to EC2:1 and result for EC2:2.*

	EC2:1 Damage [ - ]	EC2:2
Shear reinforcement	8,723	NOT OK

The shear reinforcement amount was increased due to the insufficient fatigue life, see Table 6.6 According to the first method the required amount of shear reinforcement was 81 000 mm<sup>2</sup>. For this amount the second method still indicated that the fatigue life was inadequate. With a further increase to 170 200 mm<sup>2</sup> the shear reinforcement had a proper fatigue design according to both EC2 methods.

*Table 6.6 Increase of shear reinforcement amount for fatigue design, according to methods in EC2.*

	Static design	Fatigue design: EC2:1	Fatigue design: EC2:2
Reinforcement amount [mm <sup>2</sup> ]	54 329	81 000	170 200
Damage [ - ]	8,723	0,992	0,003
EC2:2	NOT OK	NOT OK	OK

## **7 Parametric Study of Foundation Slab Subjected to Fatigue**

### **7.1 Parameters studied**

The fatigue assessment of the preliminarily designed foundation slab showed more damage than acceptable for the top and shear reinforcement, while the other critical components of the slab had sufficient fatigue life. The parametric study was performed to give a deeper understanding of how fatigue affects a foundation slab subjected to cyclic loading. The study also involved a more thorough comparison between the different fatigue assessment methods.

The parametric study was carried out by assessing the designed foundation slab, if nothing else is stated, and changing the parameters affecting the fatigue life. The preliminarily designed slab has a width of 16 meters, a height of 2 meters and a concrete strength class of C30/37. Parameters that influence the fatigue life of the slab, and that were altered in this study, are the width and the height of the slab. In order to choose ranges for the parameters studied the static design was verified for all the values. For changes of the width and height the foundation stiffness in ultimate limit state is what determines the range for the variation of the parameters.

The soil pressure is dependent on the self weight of the slab and with a larger width or height the volume and self weight of the slab increases. The resulting reaction force from the ground is then acting on a larger area of the slab and the soil pressure decreases for an increased height or width. The upper limit for the soil pressure caused by the slab is set to 400 kPa, to ensure sufficient soil stability. No lower limit is set for the soil pressure.

Figure 7.1 shows the soil pressure versus width of the slab. The soil pressure is decreasing with increasing width. A width of the slab of 15 meters results in a soil pressure of 437 kPa; this is slightly above the limit for the soil pressure.

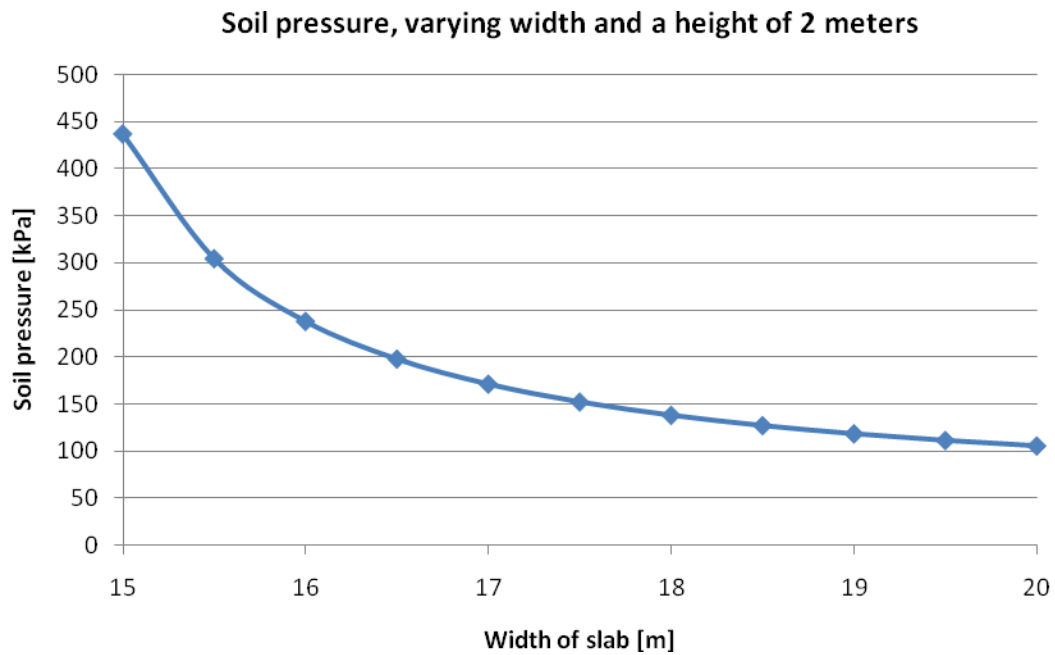


Figure 7.1 Soil pressure for a slab height of 2 meters and varying widths.

In Figure 7.2 the decrease of soil pressure with increasing height is shown. For the height 1.5 meters the soil pressure is 470 kPa and the limit of 400 kPa is exceeded.

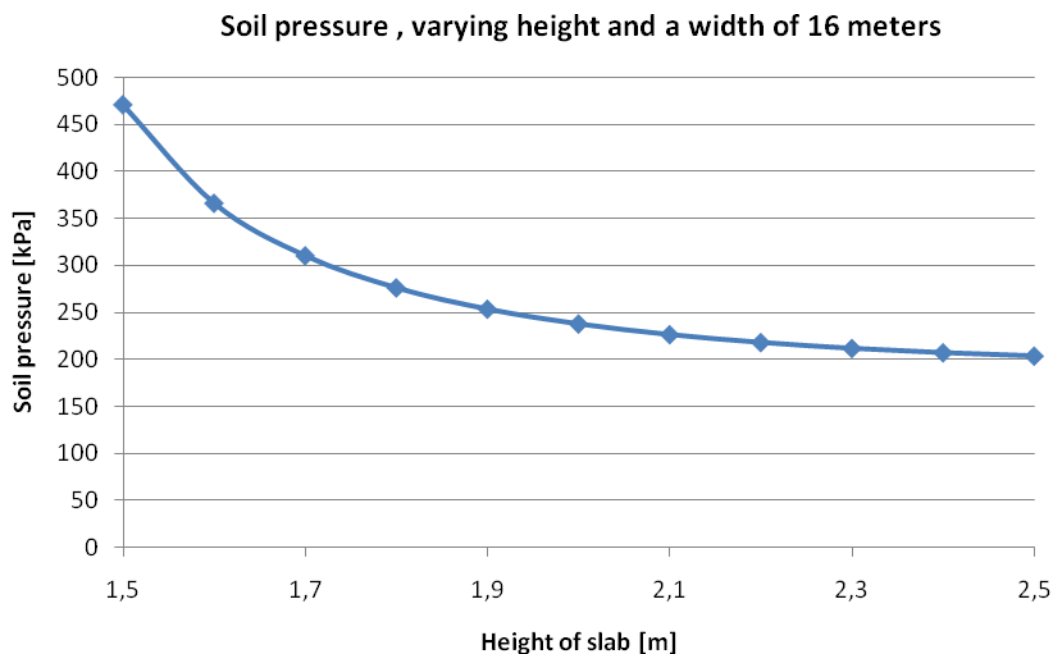


Figure 7.2 Soil pressure for a slab width of 16 meters and varying heights.

The slab also has to be verified for adequate rotational stiffness since this, as well as the soil pressure, is dependent on the width and height of the slab. The demand for rotational stiffness is, as mentioned previously, 1500 MNm/degree.

The change of slab width never results in a rotational stiffness below the limit, see Figure 7.3.

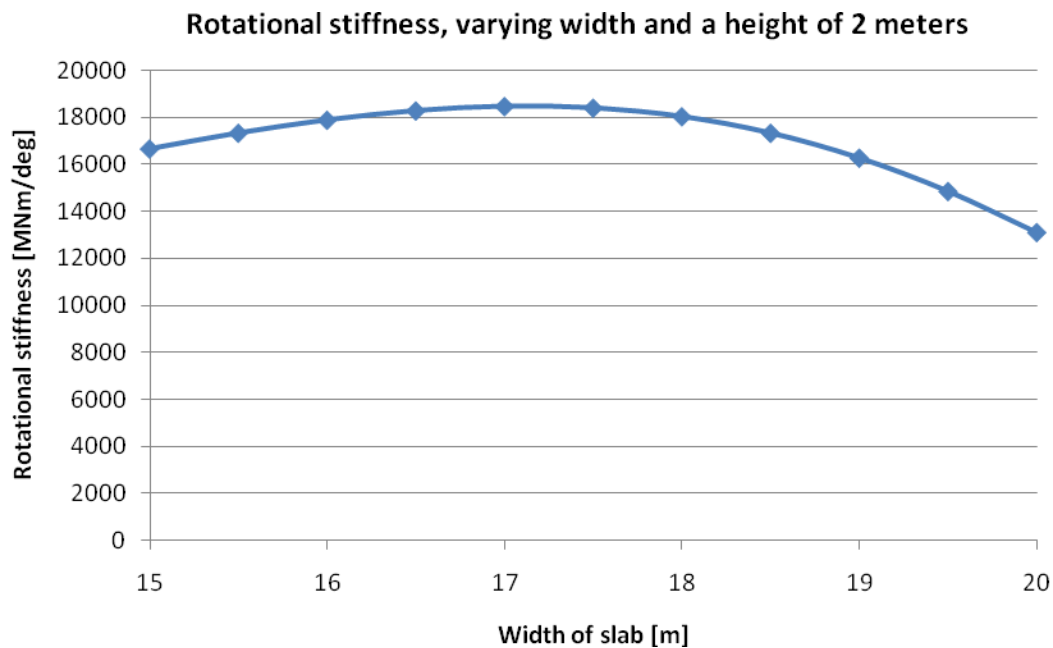


Figure 7.3 Rotational stiffness for a slab height of 2 meters and varying widths.

The rotational stiffness increases when the height is increasing, see Figure 7.4. The heights included in this study all have sufficient rotational stiffness, while it will not be possible to choose the range for the height with this check.

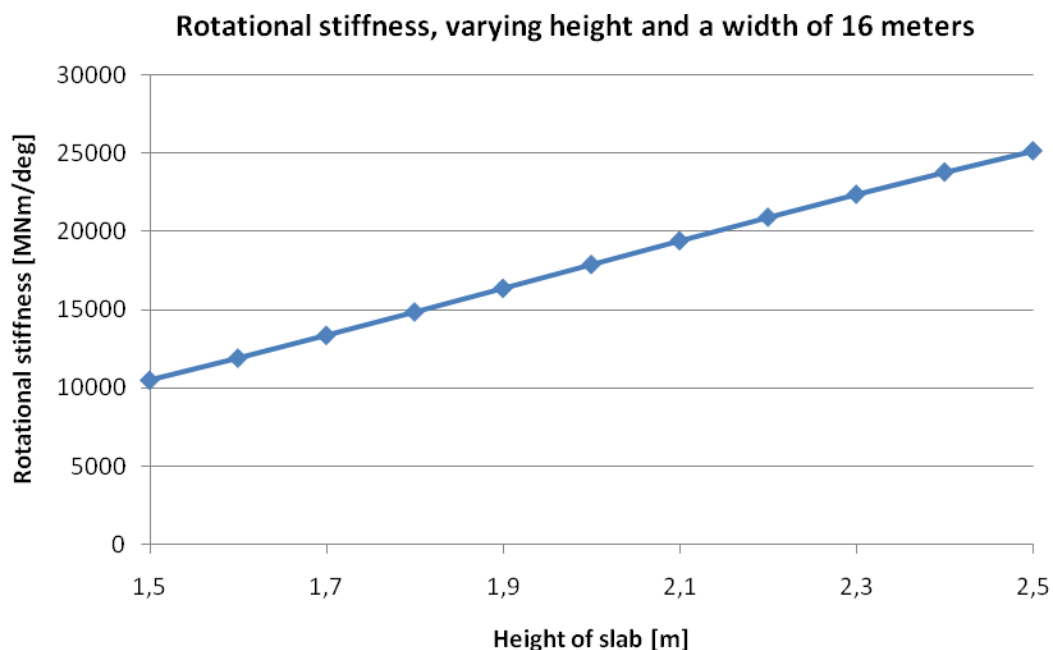


Figure 7.4 Rotational stiffness for a slab width of 16 meters and varying heights.

The range for the variation of the width of the slab was set to 15 to 20 meters. The width of 15 meter was chosen as the lower limit for the parametric study as this width results in a soil pressure above the demanded. The upper limit of 20 meters was chosen since this is within the normal range of widths for a foundation slab of a wind power plant.

The heights included in the parametric study are 1.5 to 2.5 meters. The smallest height, 1.5 meters, results in soil pressure just above the acceptable level. Since no upper limit for the height is determined by the stiffness of the slab, the limit was chosen as a reasonable maximum height for a foundation slab of a wind power plant.

In the parametric study presented below the following abbreviations will be used; Eurocode 2 will be referred to as EC2 and fib Model Code 2010 as MC2010.

## 7.2 Variation of width of slab

The study regarding variation of the width of the slab was executed for the designed slab, which has a height of 2 meters and concrete strength class C30/37. The first part of the assessment was performed for the varying widths and with reinforcement amounts that were changed in order for the static design to have sufficient flexural and shear capacity in the ultimate limit state, see Table 7.1.

*Table 7.1 Varying widths and corresponding soil pressure, rotational stiffness and reinforcement amounts that ensure adequate flexural and shear capacity in the ultimate limit state.*

Width [m]	Soil pressure [kPa]	Rotational stiffness [MNm/deg]	$A_s$ [mm <sup>2</sup> ]	$A'_s$ [mm <sup>2</sup> ]	$A_{sv}$ [mm <sup>2</sup> ]
15	436,6	16 651	79 897	18 668	56 112
15.5	304,1	17 323	78 361	18 663	55 235
16	237,5	17 877	76 812	18 621	54 329
16.5	197,5	18 271	75 248	18 544	53 395
17	170,9	18 458	73 668	18 433	52 432
17.5	151,9	18 390	72 068	18 290	51 440
18	137,8	18 023	70 450	18 116	50 419
18.5	126,8	17 320	68 810	17 912	49 370
19	118,1	16 261	67 147	17 681	48 292
19.5	111,0	14 845	65 462	17 422	47 185
20	105,2	13 095	63 751	17 137	46 049

The fatigue assessment for the varying slab widths was performed for both the concrete and the different types of reinforcement. Table 7.2 shows the damage of the concrete for the methods from EC2 and MC2010 as well as the results from the second EC2 method. As can be seen there is no or insignificant damage of the concrete according to the two damage calculation methods. The second EC2 method also shows that the concrete has an adequate fatigue life. Since there is insignificant fatigue damage of the concrete, and the results are cohesive, compression of the concrete was left without further consideration in this part of the study.

*Table 7.2 Damage of concrete according to EC2:1 and MC2010 and results for EC2:2, for the varying widths.*

Width [m]	MC2010 Damage [ - ]	EC2:1 Damage [ - ]	EC2:2
15	0	$3,12 \cdot 10^{-6}$	OK
15.5	0	$3,05 \cdot 10^{-6}$	OK
16	0	$3,77 \cdot 10^{-6}$	OK
16.5	0	$3,06 \cdot 10^{-6}$	OK
17	0	$2,73 \cdot 10^{-6}$	OK
17.5	0	$3,79 \cdot 10^{-6}$	OK
18	0	$3,75 \cdot 10^{-6}$	OK
18.5	0	$3,73 \cdot 10^{-6}$	OK
19	0	$2,74 \cdot 10^{-6}$	OK
19.5	0	$3,30 \cdot 10^{-6}$	OK
20	0	$3,75 \cdot 10^{-6}$	OK

Table 7.3 below shows the damage of the reinforcement according to EC2 and the results from the second EC2 method for fatigue assessment of the reinforcement. The results from these two methods are consistent, except for the bottom reinforcement. The second EC2 method indicates that the bottom reinforcement would have an exceeded fatigue life for widths of 15 to 18 meters. However, the damage calculation method shows a damage of just 0.022% to 0.5% for these widths. The amounts of top and shear reinforcement need to be increased for all the widths included in the study, according to both EC2 methods.



Table 7.3 Damage of the reinforcement according to EC2:1 and results for EC2:2, for the varying widths.

Width [m]	EC2:1 Damage [ - ]			EC2:2		
	$A_s$	$A'_s$	$A_{sv}$	$A_s$	$A'_s$	$A_{sv}$
15	0,005034	60,010	3,773	NOT OK	NOT OK	NOT OK
15.5	0,003691	46,415	7,257	NOT OK	NOT OK	NOT OK
16	0,002541	62,155	8,723	NOT OK	NOT OK	NOT OK
16.5	0,001638	63,362	11,246	NOT OK	NOT OK	NOT OK
17	0,000968	69,335	14,583	NOT OK	NOT OK	NOT OK
17.5	0,000504	98,647	17,089	NOT OK	NOT OK	NOT OK
18	0,000219	129,767	19,216	NOT OK	NOT OK	NOT OK
18.5	0,000031	144,264	21,631	OK	NOT OK	NOT OK
19	0,000002	156,259	24,497	OK	NOT OK	NOT OK
19.5	0	168,179	27,690	OK	NOT OK	NOT OK
20	0	182,603	31,336	OK	NOT OK	NOT OK

The fatigue assessment of the bottom reinforcement shows that the damage is constantly low. The highest damage is found for the width 15 meters, and that is estimated to 0.5 %. Figure 7.5 shows the damage of the bottom reinforcement for the varying widths. With an increasing width the fatigue damage decreases.

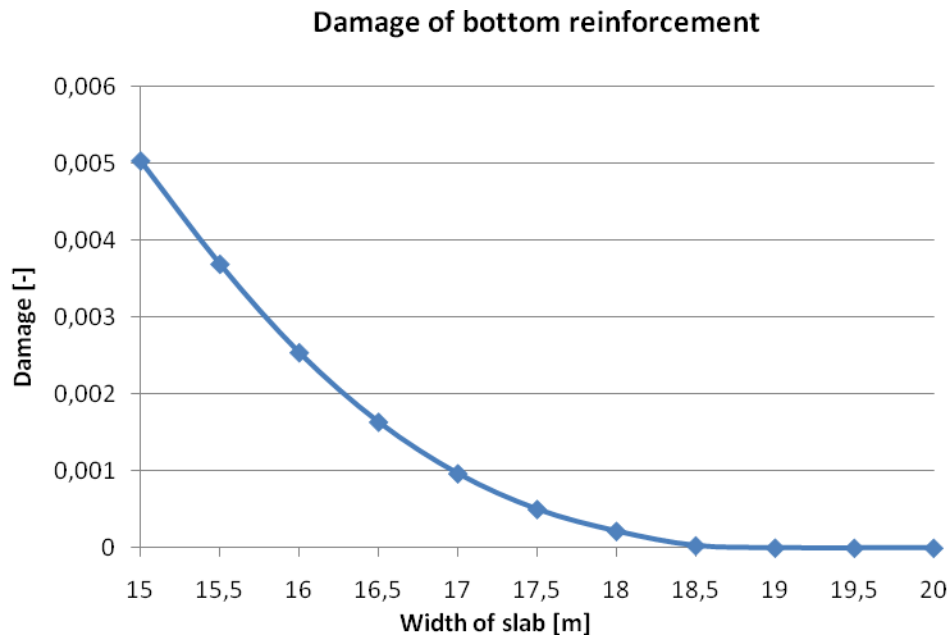


Figure 7.5 Damage of the bottom reinforcement, for varying widths, according to EC2:1.

The fatigue life of the top reinforcement is exceeded for all of the reinforcement amounts estimated for the different widths. The damage becomes significantly higher for the decreased reinforcement amounts used for the larger widths. The slab with width 20 meters has a damage of the top reinforcement of 182.6, while the 15 meter slab has a damage of the top reinforcement of 60.0. See Figure 7.6 for the relation between damage of the top reinforcement and width of slab.

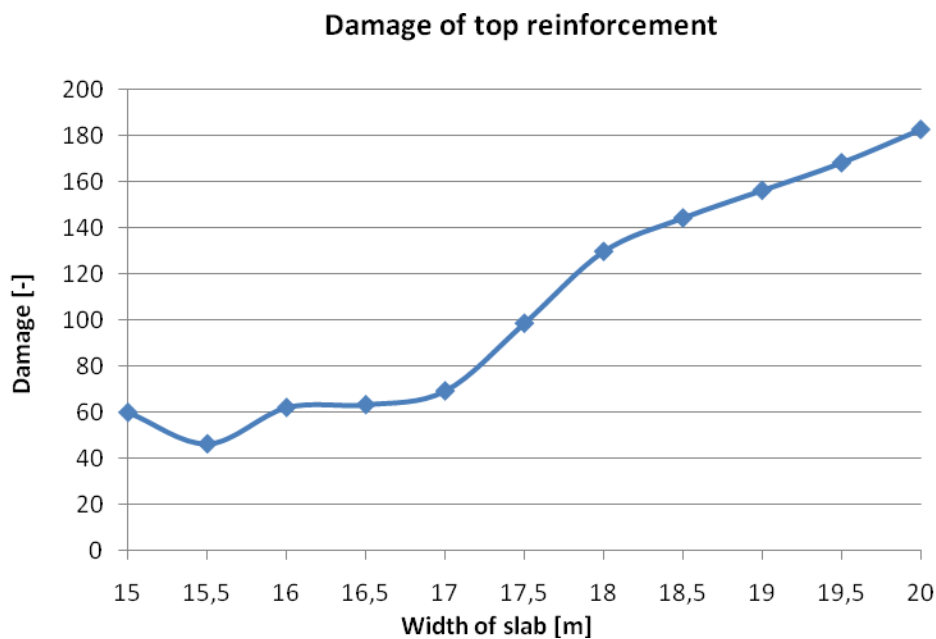


Figure 7.6 Damage of the top reinforcement, for varying widths, according to EC2:1.

The fatigue assessment of the shear reinforcement shows that the slab will have damage above one for all the different widths, which would mean that the fatigue life of the reinforcement is exceeded. In Figure 7.7 the damage development in the shear reinforcement is shown. The damage of the shear reinforcement is increasing with increasing width of slab.

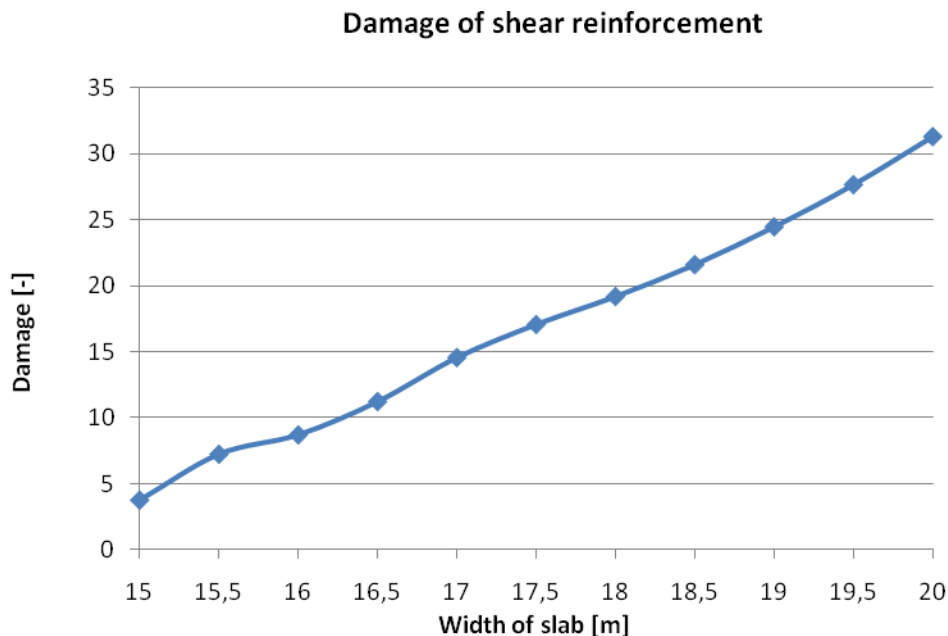


Figure 7.7 Damage of the shear reinforcement, for varying widths, according to EC2:1.

### 7.2.1 Study of damage development in top and shear reinforcement

When the first part of the parametric study regarding the slab width was performed a deeper analysis of the response of the top and shear reinforcement followed. Both the top and shear reinforcement showed an insufficient fatigue life for all the widths included in the study. Due to this the damage development in this reinforcement is of interest and it was therefore further studied.

For every full meter of the width the top reinforcement amount was increased to see the damage development with the increasing reinforcement amount, see Figure 7.8. The damage was found to decrease rapidly for an increase of the reinforcement amount up to 30 000 mm<sup>2</sup>. As the reinforcement amount was further increased the rate of the reducing damage was decreased.

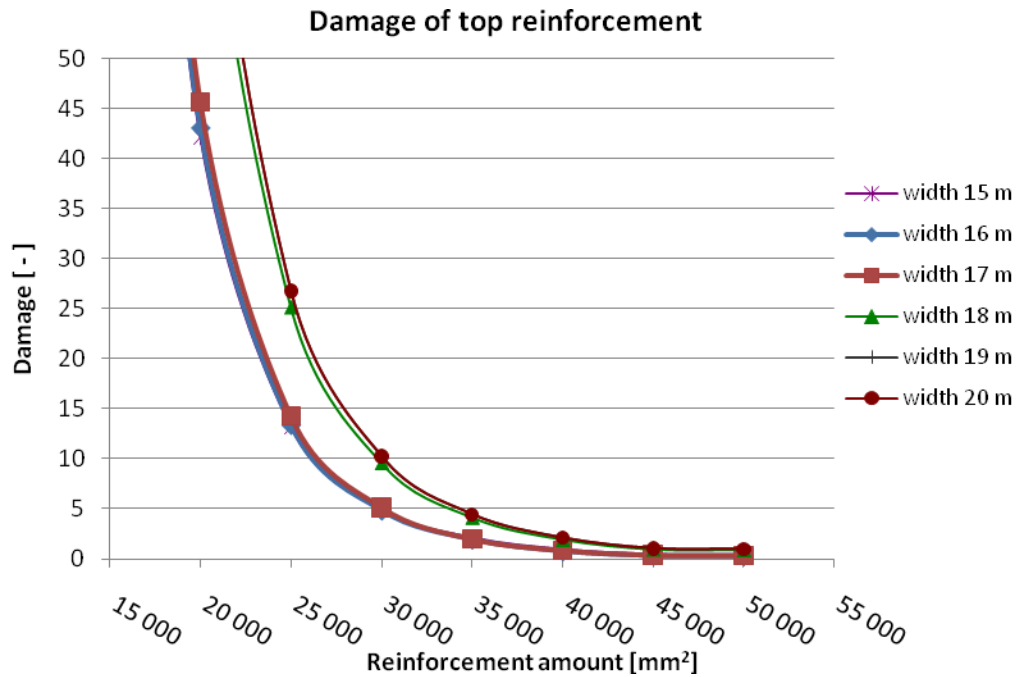


Figure 7.8 Damage of the top reinforcement for increasing reinforcement amount and varying widths of the slab.

In order for the slab to have a sufficient fatigue life the damage should be below one. The amount of top reinforcement was further increased to find the required amount for every width of the slab, see Figure 7.9.

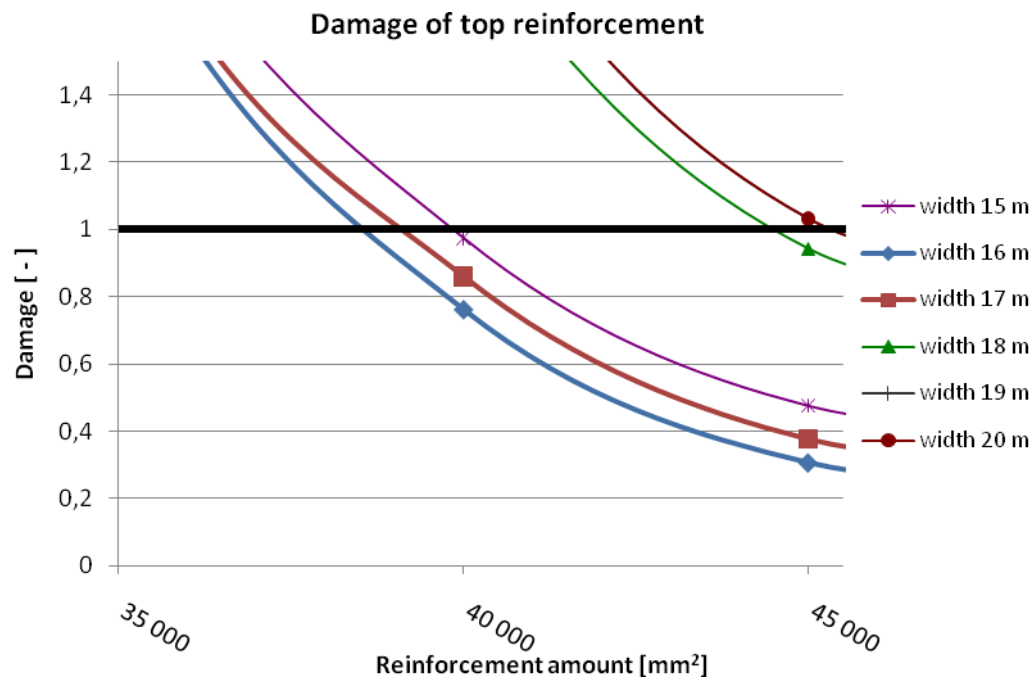


Figure 7.9 Damage of the top reinforcement and varying widths of the slab, for reinforcement amounts ensuring damage below one.

The final amounts of top reinforcement needed for the different widths, with regard to fatigue, are shown in Table 7.4. The difference between the amount needed for a width of 15 meters and the amount for a width of 20 meters is 5 400 mm<sup>2</sup>. The difference in fatigue damage for the reinforcement amounts estimated in the static design was very large, but as seen in these results the damage reduces rapidly with increasing amount of reinforcement.

*Table 7.4 Varying widths and top reinforcement amounts that result in a sufficient fatigue life according to EC2:1 and results for EC2:2.*

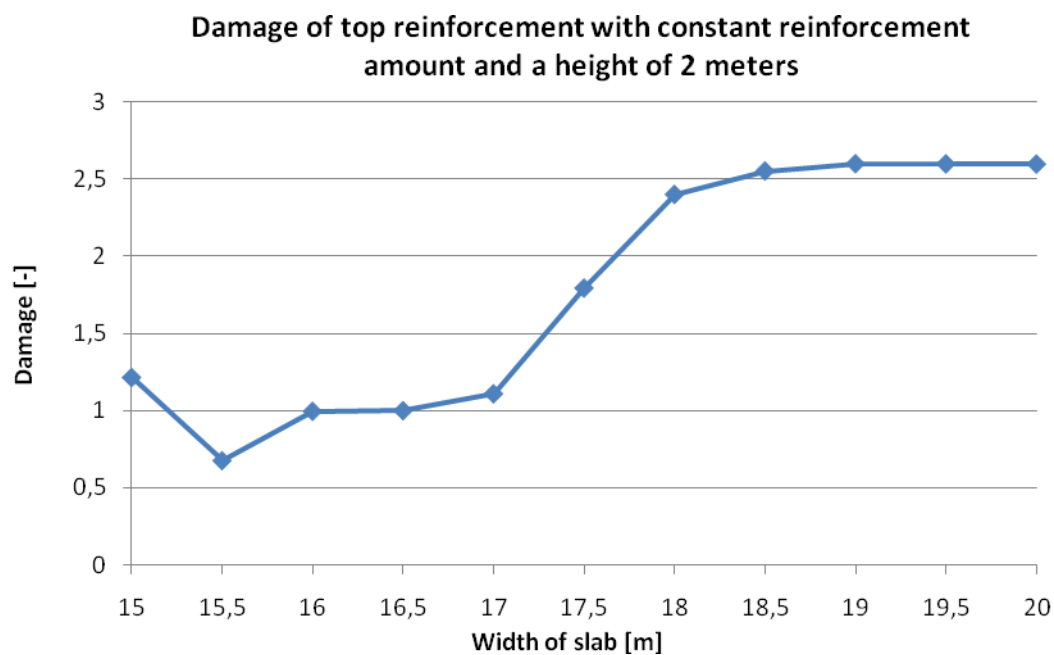
Width [m]	Top reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
15	18 668	39 900	0,989	NOT OK
16	18 621	38 500	0,997	NOT OK
17	18 433	39 200	0,988	NOT OK
18	18 116	44 600	0,998	NOT OK
19	17 681	45 300	0,991	NOT OK
20	17 137	45 300	0,991	NOT OK

Table 7.5 shows the top reinforcement amounts needed for sufficient fatigue capacity according to the second EC2 method. As seen in the table these reinforcement amounts are considerably larger than the ones estimated by the damage calculation method. It is also noticeable that EC2:1 indicates that the needed amount of top reinforcement increases with 5 400 mm<sup>2</sup> when changing the width from 15 to 20 meters. EC2:2 however, shows that the same change of width would result in an increase of top reinforcement of 27 300 mm<sup>2</sup>.

*Table 7.5 Varying widths and top reinforcement amounts that result in a sufficient fatigue life according to EC2:2 and damage according to EC2:1.*

Width [m]	Top reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
15	18 668	55 000	0,116	OK
16	18 621	51 900	0,086	OK
17	18 433	65 700	0,019	OK
18	18 116	76 700	0,015	OK
19	17 681	82 300	0,011	OK
20	17 137	82 300	0,011	OK

In order to try to isolate the parameters affecting the fatigue damage the amount of top reinforcement was set to 38 500 mm<sup>2</sup>, which is the amount estimated in the fatigue assessment (EC2:1) of the preliminarily designed foundation slab. The slab height is also set as a constant of 2 meters. Figure 7.10 shows the damage development in the top reinforcement for constant reinforcement amount and height of slab.



*Figure 7.10 Damage of the top reinforcement according to EC2:1, for varying widths and with constant reinforcement amount and height of slab.*

When the analysis of the top reinforcement was done the same analysis was performed for the shear reinforcement in order to see if the damage development is similar. Figure 7.11 shows the development of damage in the shear reinforcement, for increasing reinforcement amount and varying widths. The damage reduces with the increasing reinforcement amount, but with a slower pace than the damage of the top reinforcement.

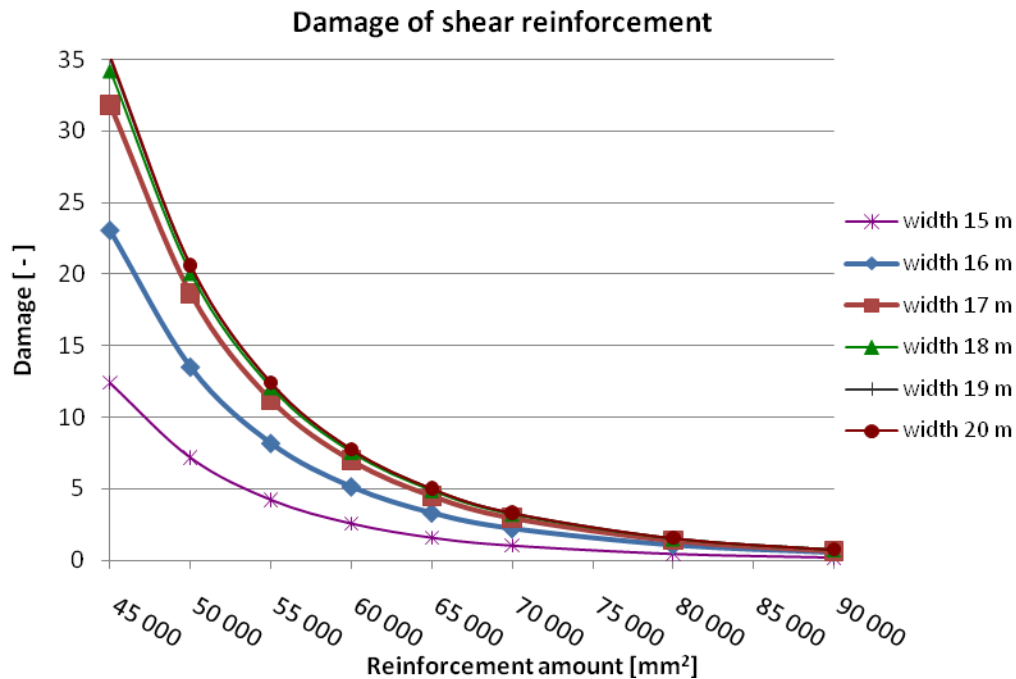


Figure 7.11 Damage of the shear reinforcement for increasing reinforcement amount and varying widths of the slab.

The reinforcement amount was increased until the damage was below one, and the fatigue life sufficient, for all the widths. The result is shown in Figure 7.12.

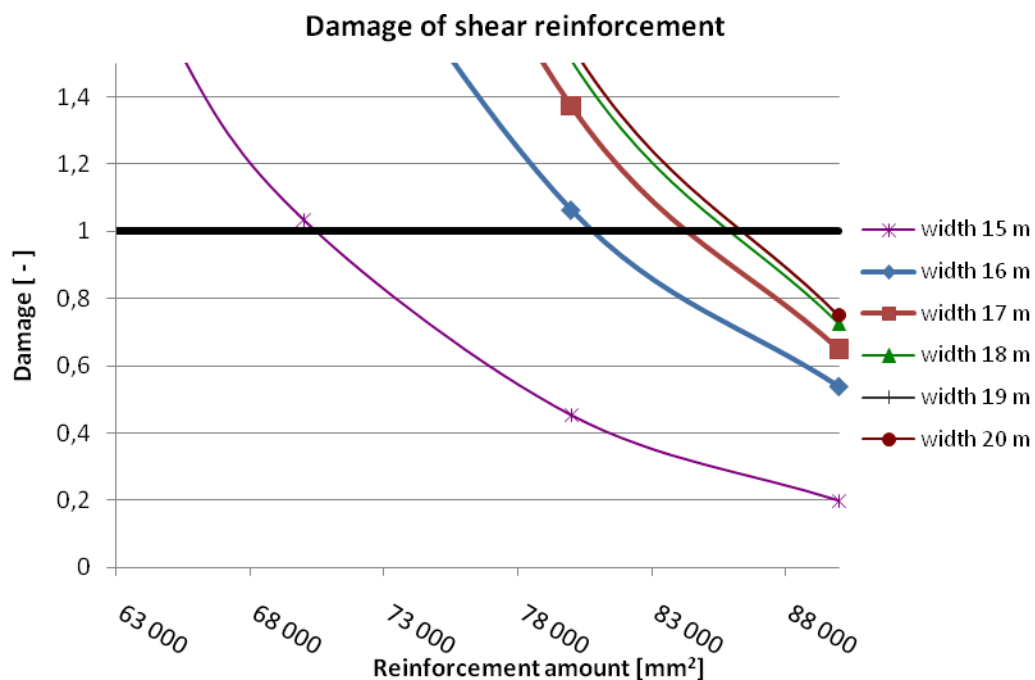


Figure 7.12 Damage of the shear reinforcement and varying widths of the slab, for reinforcement amounts ensuring damage below one.

The amounts of shear reinforcement needed according to the first EC2 method are shown in Table 7.6, together with the result from the second method. The required reinforcement amount is decreasing with increasing width of slab. As seen in the table the second method indicates an insufficient fatigue life, while the damage according to the first method is below one.



*Table 7.6 Varying widths and shear reinforcement amounts that result in a sufficient fatigue life according to EC2:1 and results for EC2:2.*

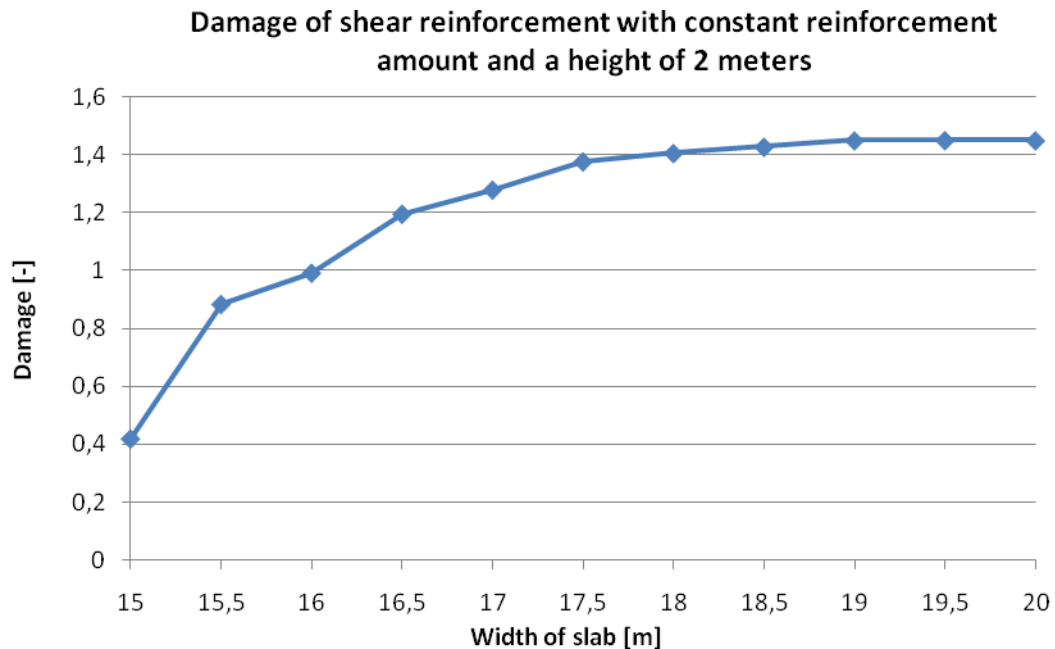
Width [m]	Shear reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
15	56 112	70 500	0,993	NOT OK
16	54 329	81 000	0,992	NOT OK
17	52 432	84 200	0,996	NOT OK
18	50 419	85 500	0,998	NOT OK
19	48 292	86 000	0,994	NOT OK
20	46 049	86 000	0,994	NOT OK

To further compare the two methods the needed amount of shear reinforcement according to the second method was estimated, see Table 7.7. These amounts are considerably larger than the amounts estimated by the damage calculation method. It can also be seen that EC2:2 indicate that the required amount of shear reinforcement decreases with increasing width, which is the opposite to the result of EC2:1.

*Table 7.7 Varying widths and shear reinforcement amounts that result in a sufficient fatigue life according to EC2:2 and damage according to EC2:1.*

Width [m]	Shear reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
15	56 112	181 400	0,001	OK
16	54 329	170 200	0,003	OK
17	52 432	158 200	0,006	OK
18	50 419	145 600	0,018	OK
19	48 292	156 300	0,011	OK
20	46 049	156 300	0,011	OK

The amount of shear reinforcement was set to  $81\,000\text{ mm}^2$ , which was the required amount according to the fatigue verification of the preliminarily designed slab. The height of the slab was kept constant at 2 meters and then the damage for the different widths was calculated, see Figure 7.13. The damage increases with increasing width up to 17.5 meters, then the rate of the increase of damage decreases.



*Figure 7.13 Damage of the shear reinforcement according to EC2:1, for varying widths and with constant reinforcement amount and height of slab.*

### 7.3 Variation of height of slab

The parametric study regarding variation of the height of the slab was performed for a slab with width and concrete strength class as for the preliminarily designed slab, which means a width of 16 meters and strength class C30/37. The first part of the fatigue assessment of the varying height was performed for the different heights with reinforcement amounts changed for sufficient static capacity of the slab, see Table 7.8.

*Table 7.8 Varying heights and corresponding soil pressure, rotational stiffness and reinforcement amounts that ensure adequate flexural and shear capacity in the ultimate limit state.*

Height [m]	Soil pressure [kPa]	Rotational stiffness [MNm/deg]	$A_s$ [mm <sup>2</sup> ]	$A'_s$ [mm <sup>2</sup> ]	$A_{sv}$ [mm <sup>2</sup> ]
1,5	471,0	10 496	114 631	25 474	57 617
1,6	366,1	11 904	104 828	23 791	56 959
1,7	310,1	13 358	96 374	22 291	56 302
1,8	275,9	14 847	89 010	20 944	55 644
1,9	253,3	16 357	82 540	19 727	54 987
2	237,5	17 877	76 812	18 621	54 329
2,1	226,1	19 392	71 707	17 612	53 672
2,2	217,7	20 890	67 129	16 687	53 014
2,3	211,4	22 355	63 000	15 836	52 357
2,4	206,8	23 772	59 260	15 050	51 699
2,5	203,3	25 127	55 855	14 322	51 042

With the reinforcement amounts as shown in the table above the fatigue assessment of the different slab heights was executed. Table 7.9 shows the results for both the EC2 and MC2010 methods regarding fatigue of the concrete. As can be seen the damage is insignificant for both damage calculation methods and the second EC2 method also indicates a sufficient fatigue life of the concrete.

*Table 7.9 Damage of concrete according to EC2:1 and MC2010 and results for EC2:2, for the varying heights.*

Height [m]	MC2010 Damage [ - ]	EC2:1 Damage [ - ]	EC2:2
1,5	$1,58 \cdot 10^{-8}$	$9,96 \cdot 10^{-6}$	OK
1,6	$5,41 \cdot 10^{-10}$	$4,65 \cdot 10^{-6}$	OK
1,7	$7,00 \cdot 10^{-12}$	$3,35 \cdot 10^{-6}$	OK
1,8	0	$2,90 \cdot 10^{-6}$	OK
1,9	0	$3,24 \cdot 10^{-6}$	OK
2	0	$3,77 \cdot 10^{-6}$	OK
2,1	0	$3,74 \cdot 10^{-6}$	OK
2,2	0	$3,05 \cdot 10^{-6}$	OK
2,3	0	$3,05 \cdot 10^{-6}$	OK
2,4	0	$2,70 \cdot 10^{-6}$	OK
2,5	0	$2,35 \cdot 10^{-6}$	OK

The MC2010 method shows a small damage for heights 1.5 to 1.7 meters, while the EC2 method shows a slightly larger damage for all the heights included in the study. Figure 7.14 shows the damage of the compressed concrete according to the methods from EC2 and MC2010.

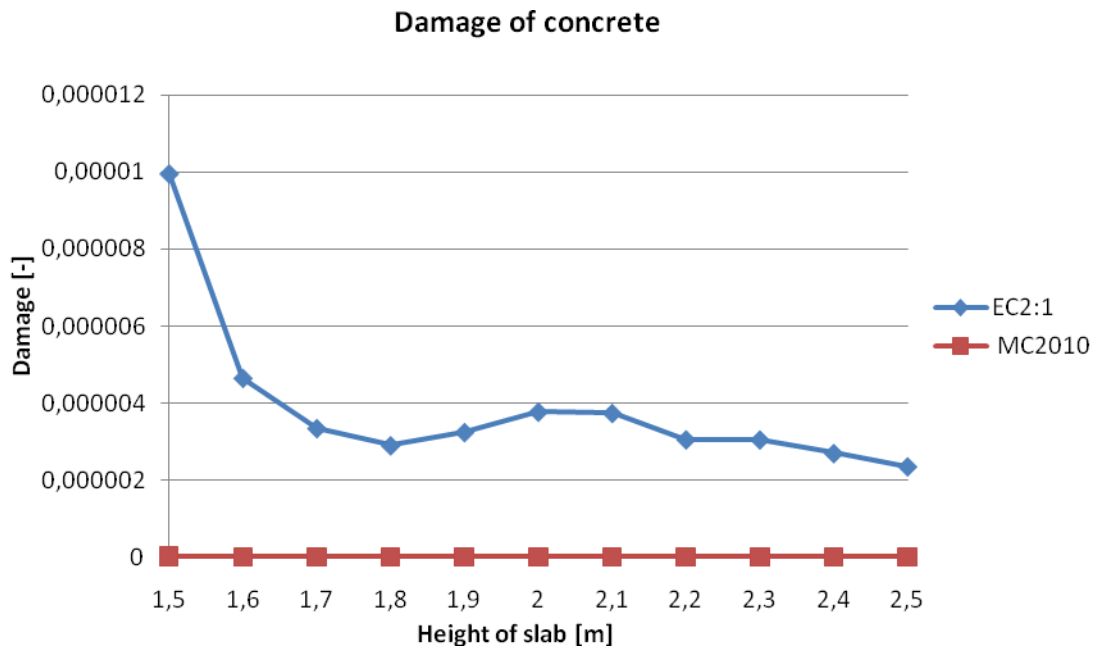


Figure 7.14 Damage of the concrete, for varying heights, according to EC2:1 and MC2010.

The different types of reinforcement were assessed with the two Eurocode methods and these results are shown in Table 7.10. The bottom reinforcement was found to have insignificant damage for all the heights, but the second method still indicated an insufficient fatigue life. The fatigue assessment regarding top and shear reinforcement showed that for all the different heights the fatigue life of this reinforcement was exceeded. The result of the second EC2 method agreed with the result of the first one, and also indicated that an increase of the top and shear reinforcement is needed to ensure a 20 year fatigue life.

Table 7.10 Damage of the reinforcement according to EC2:1 and results for EC2:2, for the varying heights.

Height [m]	EC2:1 Damage [ - ]			EC2:2		
	$A_s$	$A'_s$	$A_{sv}$	$A_s$	$A'_s$	$A_{sv}$
1,5	0,0067	62,813	2,591	NOT OK	NOT OK	NOT OK
1,6	0,0056	55,119	3,146	NOT OK	NOT OK	NOT OK
1,7	0,0047	58,911	4,548	NOT OK	NOT OK	NOT OK
1,8	0,0039	48,947	6,360	NOT OK	NOT OK	NOT OK
1,9	0,0032	55,728	7,431	NOT OK	NOT OK	NOT OK
2	0,0025	62,155	8,723	NOT OK	NOT OK	NOT OK
2,1	0,0020	55,516	10,947	NOT OK	NOT OK	NOT OK
2,2	0,0016	61,468	11,655	NOT OK	NOT OK	NOT OK
2,3	0,0012	66,335	13,366	NOT OK	NOT OK	NOT OK
2,4	0,0009	65,495	16,652	NOT OK	NOT OK	NOT OK
2,5	0,0006	73,320	18,036	NOT OK	NOT OK	NOT OK

In Figure 7.15 the result of the fatigue verification regarding the bottom reinforcement is shown. The damage of the bottom reinforcement is constantly low; it varies between 0.1 and 0.7 %. This shows that the fatigue life is sufficient for all the heights, even though the amount of bottom reinforcement for the 2.5 meter deep slab is 58 000 mm<sup>2</sup> less than the amount for the 1.5 meter deep slab.

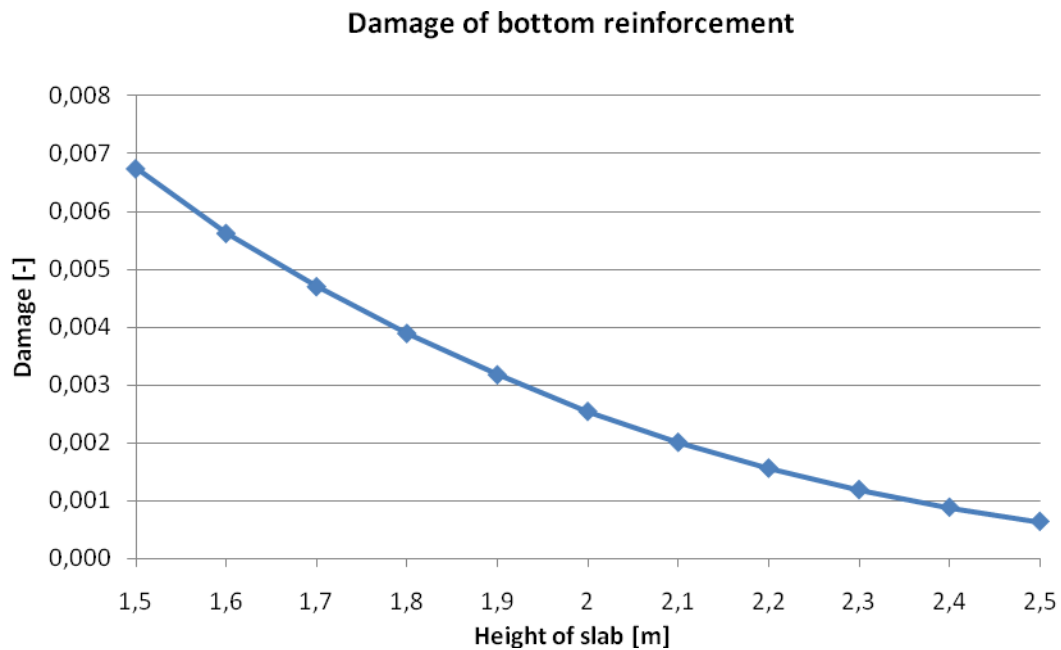


Figure 7.15 Damage of the bottom reinforcement, for varying heights, according to EC2:1.

The damage of the top reinforcement versus height is shown in Figure 7.16 and shows an insufficient fatigue life for all the heights. This was expected since the top reinforcement amount was increased in the fatigue assessment of the 16 meter wide slab; see Chapter 6 in particular Section 6.4.2. The maximum amount of top reinforcement used in this study was still lower than the amount needed according to the previous assessment.

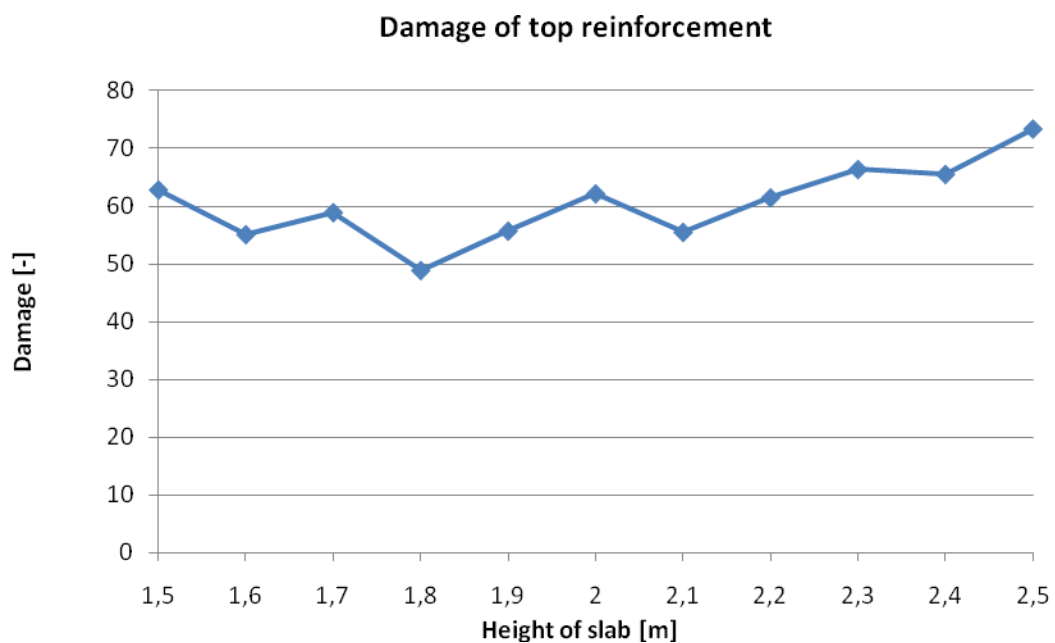


Figure 7.16 Damage of the top reinforcement, for varying heights, according to EC2:1.

When assessing the shear reinforcement with regard to fatigue and varying height, the results showed that for all the heights the fatigue life of the shear reinforcement was inadequate, see Figure 7.17. With increasing slab height the damage of the shear reinforcement increased.

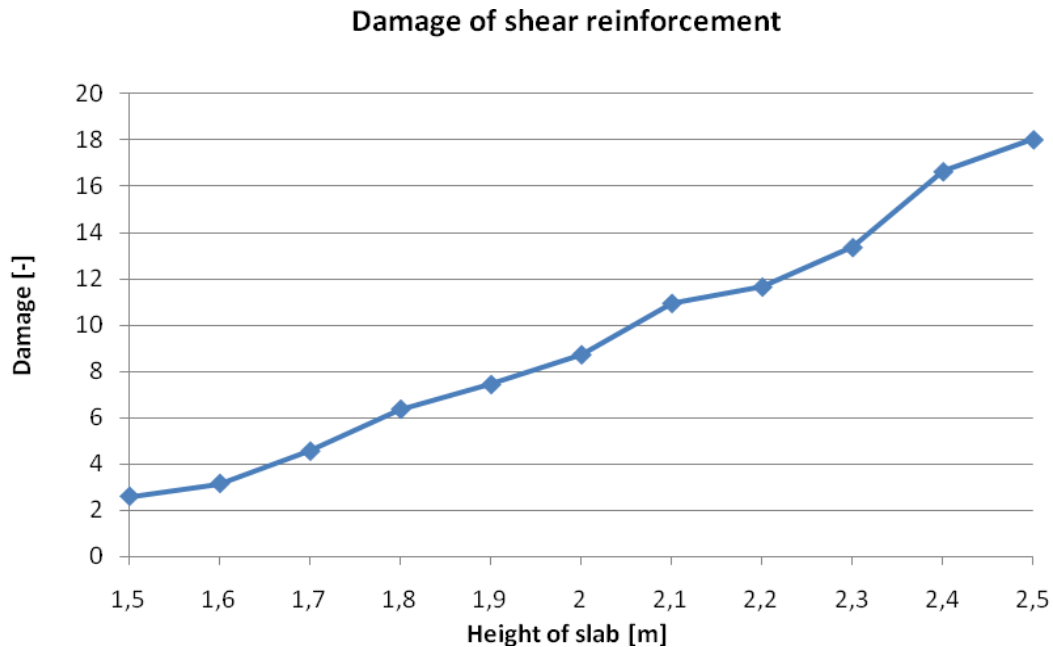


Figure 7.17 Damage of the shear reinforcement, for varying heights, according to EC2:1.

### 7.3.1 Study of damage development in top and shear reinforcement

The second part of the parametric study regarding the height of the slab consisted of a more thorough analysis of the damage development in the top and shear reinforcement. For six of the different heights the amount of top and shear reinforcement was increased to see how the damage changes and the required amounts of reinforcement were found according to both the fatigue assessment methods.

Figure 7.18 shows the damage development with increasing amount of top reinforcement. The damage decreases considerably at first, then the rate decreases.



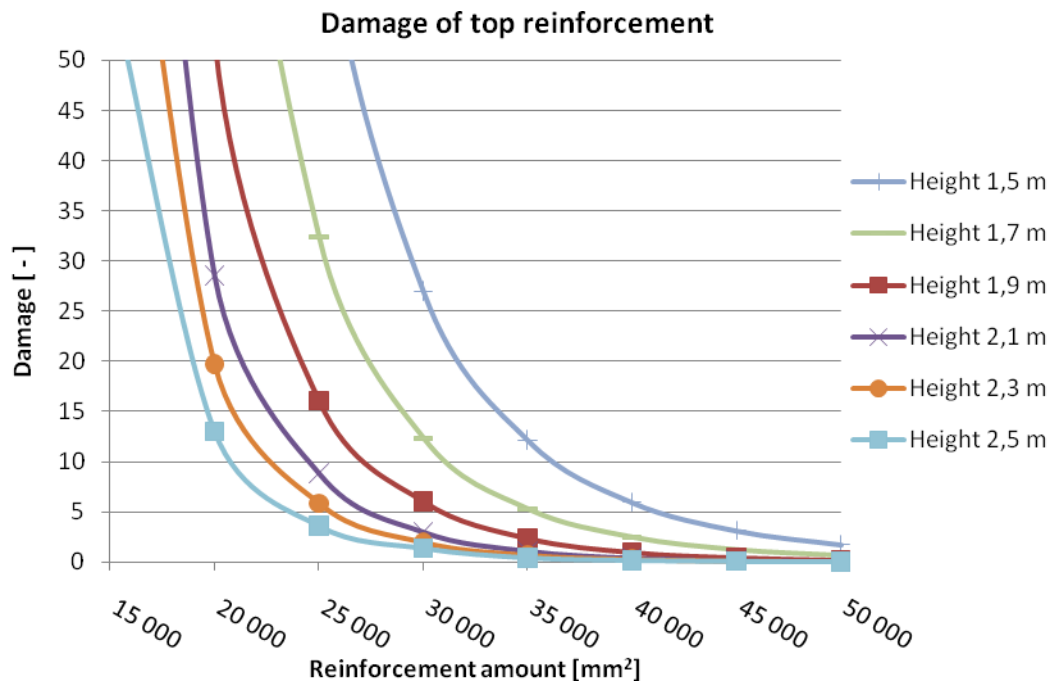


Figure 7.18 Damage of the top reinforcement for increasing reinforcement amount and varying heights of the slab.

To further study the damage development in the top reinforcement with increasing height the reinforcement amount was increased until the damage became below one, and a sufficient fatigue life was reached, see Figure 7.19.

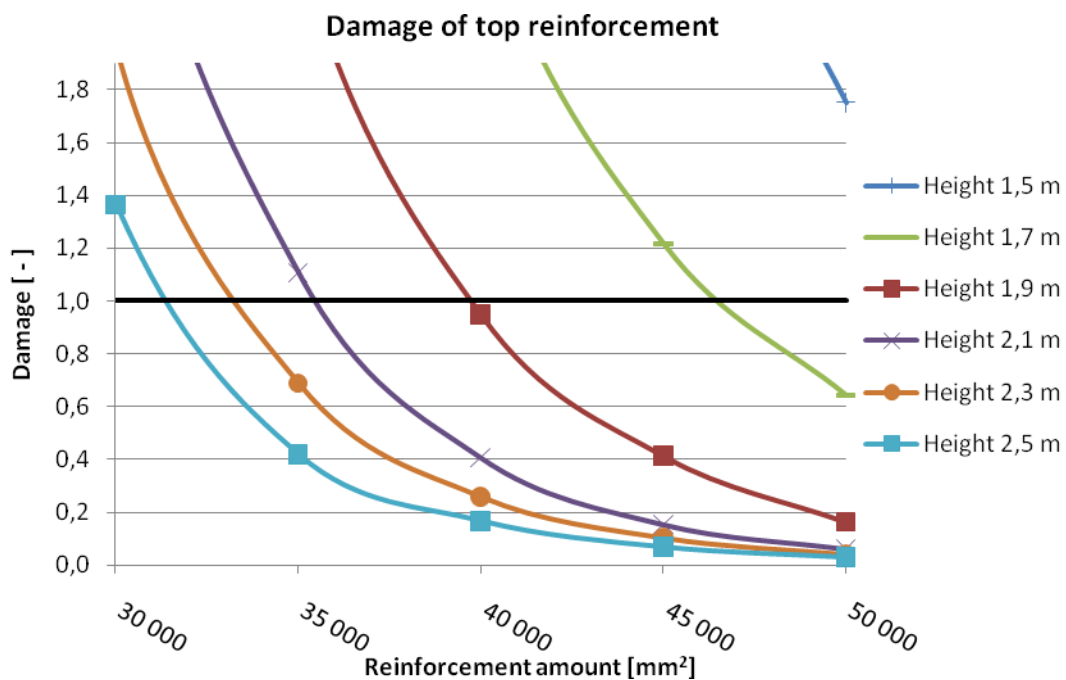


Figure 7.19 Damage of the top reinforcement and varying heights of the slab, for reinforcement amounts ensuring damage below one.

In order to show the differences between the two fatigue assessment methods for the reinforcement, the needed amount of top reinforcement was estimated according to both the methods, see Table 7.11 and 7.12.

*Table 7.11 Varying heights and top reinforcement amounts that result in a sufficient fatigue life according to EC2:1 and results for EC2:2.*

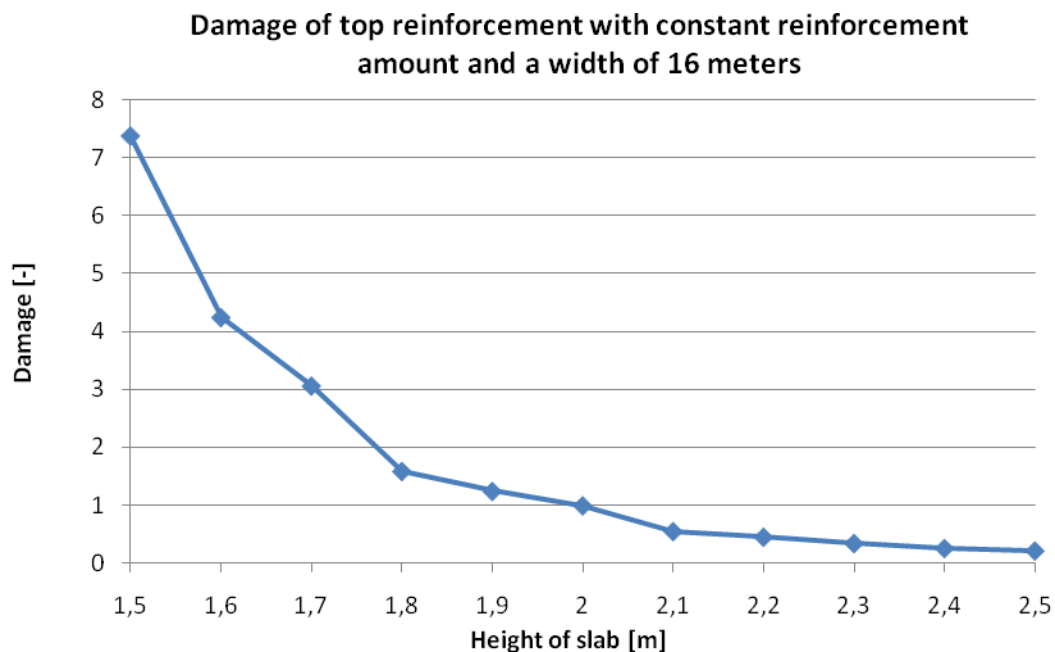
Height [m]	Top reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
1,5	25 474	55 100	0,993	NOT OK
1,7	22 291	46 500	0,993	NOT OK
1,9	19 727	39 800	0,982	NOT OK
2,1	17 612	35 600	0,986	NOT OK
2,3	15 836	33 300	0,982	NOT OK
2,5	14 322	30 800	0,996	NOT OK

*Table 7.12 Varying heights and top reinforcement amounts that result in a sufficient fatigue life according to EC2:2 and damage according to EC2:1.*

Height [m]	Top reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
1,5	25 474	75 400	0,126	OK
1,7	22 291	60 900	0,168	OK
1,9	19 727	53 700	0,086	OK
2,1	17 612	51 600	0,046	OK
2,3	15 836	51 100	0,034	OK
2,5	14 322	50 700	0,026	OK

As can be seen in the tables the amount of top reinforcement needed according to the second method is significantly larger than the amount needed according to the first method. The difference between the needed amounts for a 1.5 and a 2.5 meter slab is however about 25 000 mm<sup>2</sup> according to both the methods.

The damage of the top reinforcement was also studied by setting the amount of reinforcement to 38 500 mm<sup>2</sup>, since this is the estimated amount needed for a sufficient fatigue life of a slab with a height of 2 meters. Figure 7.20 shows the damage development of the top reinforcement for this reinforcement amount. The damage is below one for slab heights of 2 to 2.5 meters. The damage is gradually decreasing, rapidly for the smaller slab heights and slower for the larger heights.



*Figure 7.20 Damage of the top reinforcement according to EC2:1, for varying heights and with constant reinforcement amount and width of slab.*

The same type of study as for the top reinforcement was performed for the shear reinforcement. Figure 7.21 shows the damage development in the shear reinforcement for increasing reinforcement amount.

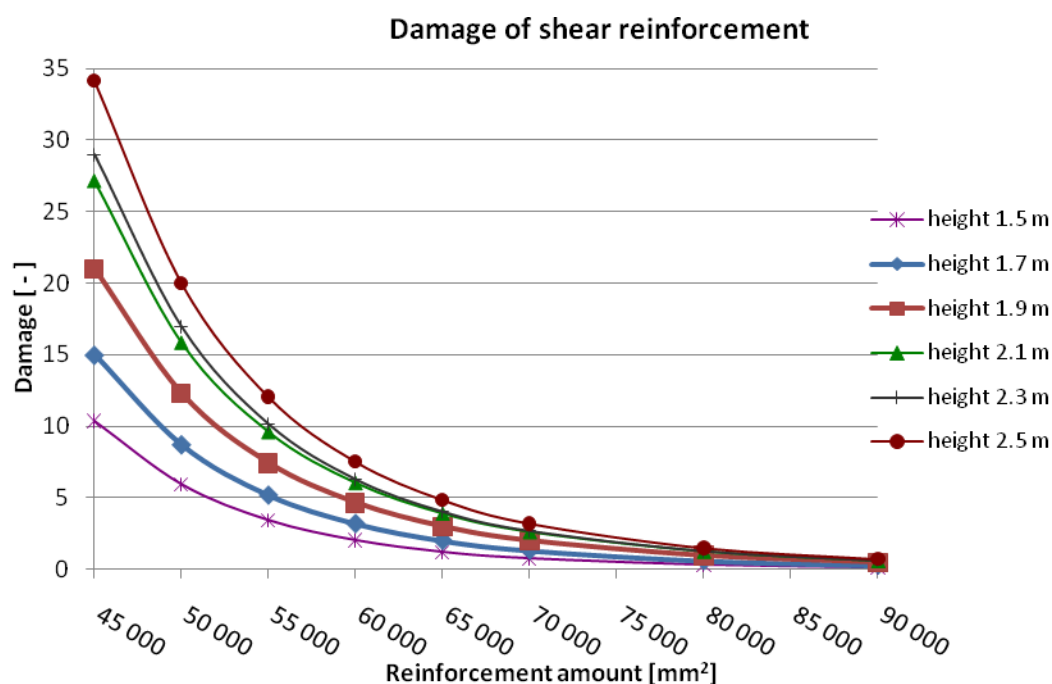


Figure 7.21 Damage of the shear reinforcement for increasing reinforcement amount and varying heights of the slab.

Figure 7.22 shows the damage development in the shear reinforcement for reinforcement amounts ensuring a sufficient fatigue life and damage below one. Here it can be seen that the damage decreases slowly when it is close to one.

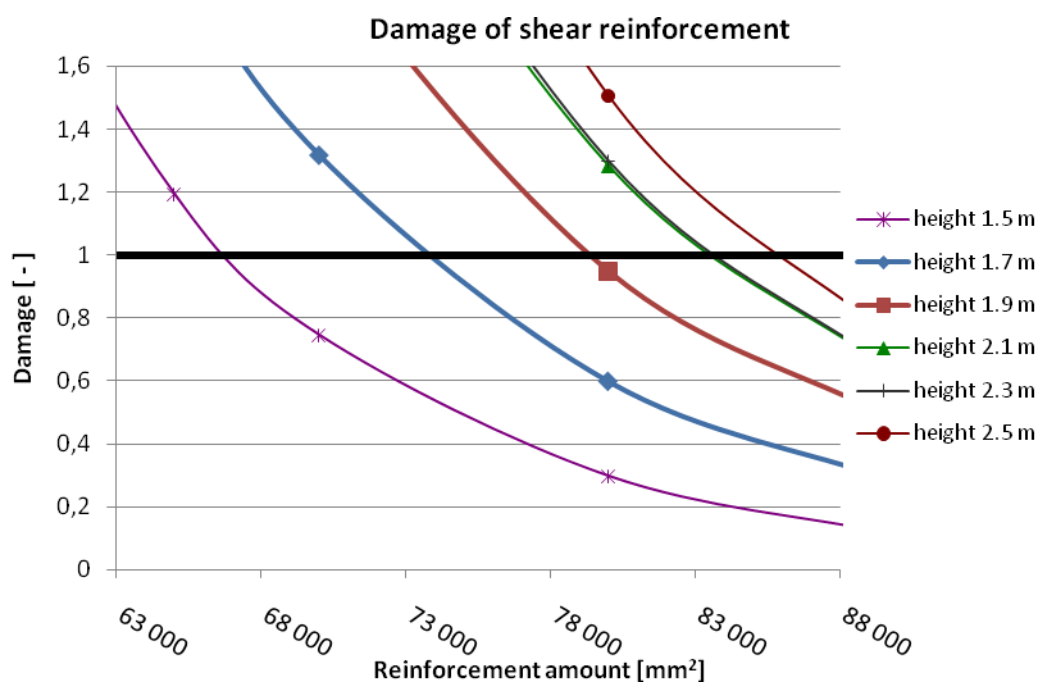


Figure 7.22 Damage of the shear reinforcement and varying heights of the slab, for reinforcement amounts ensuring damage below one.

The two EC2 methods for fatigue assessment of the reinforcement were compared by estimating the required amount of shear reinforcement according to each of the methods. The results are shown in Tables 7.13 and 7.14. As well as for the top reinforcement the second EC2 method indicates that a significantly larger amount of reinforcement is needed.

*Table 7.13 Varying heights and shear reinforcement amounts that result in a sufficient fatigue life according to EC2:1 and results for EC2:2.*

Height [m]	Shear reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
1,5	57 617	66 900	0,991	NOT OK
1,7	56 302	73 600	0,998	NOT OK
1,9	54 987	79 300	0,996	NOT OK
2,1	53 672	83 400	0,996	NOT OK
2,3	52 357	83 600	0,991	NOT OK
2,5	51 042	85 500	0,996	NOT OK

*Table 7.14 Varying heights and shear reinforcement amounts that result in a sufficient fatigue life according to EC2:2 and damage according to EC2:1.*

Height [m]	Shear reinforcement amount [mm <sup>2</sup> ]		EC2:1 Damage [ - ]	EC2:2
	Static	Fatigue		
1,5	57 617	193 400	0,0003	OK
1,7	56 302	184 100	0,001	OK
1,9	54 987	174 800	0,002	OK
2,1	53 672	165 600	0,004	OK
2,3	52 357	156 300	0,007	OK
2,5	51 042	147 000	0,017	OK

An interesting result from this comparison is that for the first EC2 method the needed amount of shear reinforcement is increasing with increasing height, and the second method shows the opposite; a decrease of shear reinforcement for increasing height. This difference between the two methods was also seen in the parametric study regarding the width of the slab.

The last part of the study of the damage development in the shear reinforcement was done by setting the reinforcement amount to  $81\,000\text{ mm}^2$ , which was the amount needed for the 16 meter wide and 2 meter high slab. The damage for this reinforcement amount is estimated for all the heights included in the study. The result is shown in Figure 7.23.

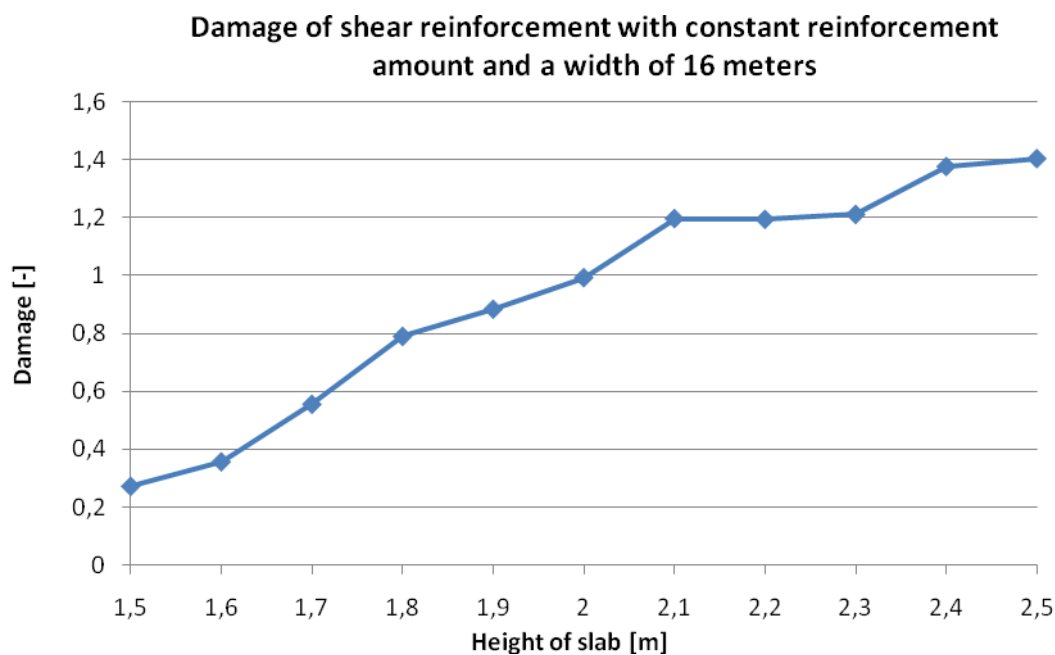


Figure 7.23 Damage of the shear reinforcement according to EC2:1, for varying heights and with constant reinforcement amount and width of slab.

## 7.4 Analysis of the results from the parametric study

To determine which structural model to use for the fatigue calculations the distance between the edge of the slab and the reaction force from the ground is used. Different models, see Section 6.2 and Figures 6.3-6.6, can be used for the maximum and minimum values of a force. For some values of the overturning moment this is necessary, especially if there is a large difference between the maximum and the minimum values. There are however occasions in the parametric study when the difference between the maximum and minimum values of the overturning moment is small, but different models are still used since the reaction force is acting close to the limit between two models. The effect of this can be observed in the diagrams from the parametric study and this is the reason why the curve does not always follow a trend.

Figure 7.24 shows the S-N curve for steel. In the figure the stress ranges in the top reinforcement are plotted against the number of cycles for the fatigue loads. As can be seen there are values above the S-N curve. This implies that for each of these values the estimated damage is above one. The amount of top reinforcement was increased until the stress ranges were decreased enough for sufficient fatigue resistance. In the figure it is seen that the values for the increased reinforcement amount are all below the S-N curve.

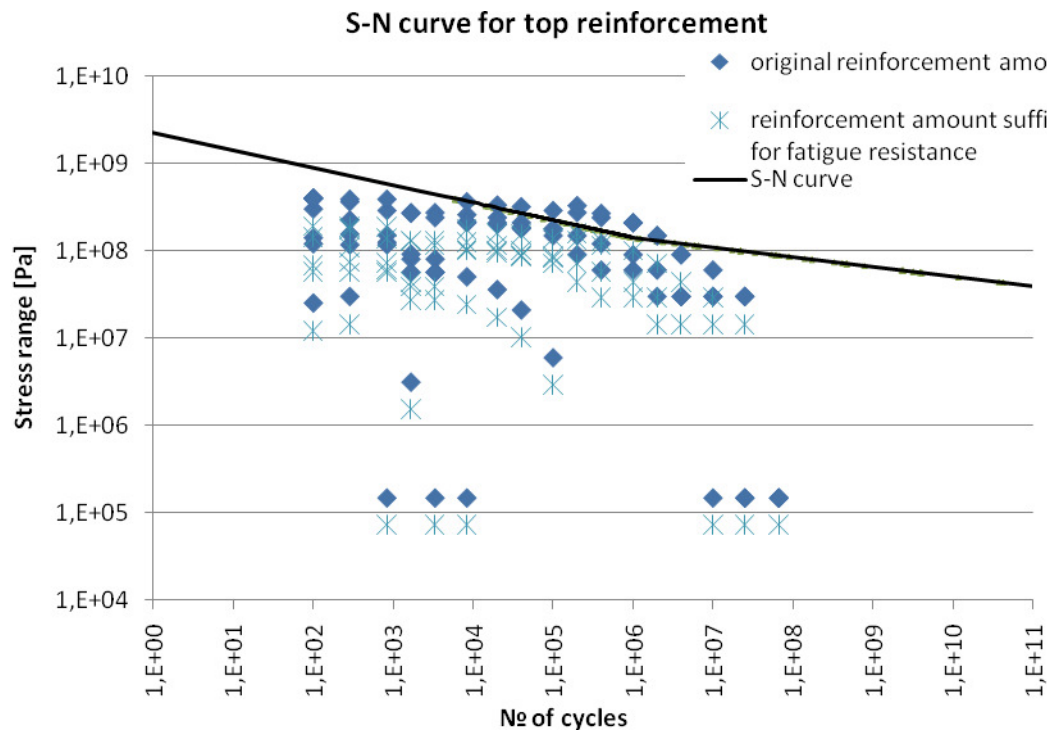


Figure 7.24 S-N curve for steel and stress ranges in the top reinforcement versus number of cycles for the fatigue load.

### 7.4.1 Analysis of the results regarding reinforcement

The damage of the bottom reinforcement was found to decrease with increasing slab width and height. When analysing these results it is seen that the reason for this behaviour is that with the smaller widths and heights strut and tie models 3 and 4 are used to a larger extent and these results in higher stresses in the bottom reinforcement. For the larger widths and heights strut and tie models 1 and 2 are used more. The choice of strut and tie model is, as mentioned earlier, dependent on where the reaction force from the ground is acting. For larger widths and heights the self weight of the slab is increasing and this is causing the reaction force to act closer to the middle of the slab and inside the anchor ring. When the reaction force is acting inside the anchor ring, there are no or small tensile forces in the bottom of the slab. Hence, the decrease of damage of the bottom reinforcement with increasing width or height of the slab is due to the increased self weight of the slab being favourable.

The fatigue damage of both shear and top reinforcement was found to be high and to increase with increasing slab width and height. For the top reinforcement the damage rapidly decreases with increasing reinforcement amount and the needed amount of reinforcement is within reasonable limits. For the shear reinforcement the decrease rate of the fatigue damage slows down before the damage is below one and the needed amount of reinforcement is therefore great. For one of the fatigue assessment methods the amount of shear reinforcement needed is even impossible to fit into the slab. For the damage calculation method in EC2 the amount of shear reinforcement needed for sufficient fatigue design is however reasonable and therefore it can be assumed that even if the initially calculated fatigue damage is high it is also reasonable. With increasing slab width and height the self weight of the slab is increased and this is unfavourable for fatigue of the top and shear reinforcement. When looking at the strut and tie models used it is seen that with increasing self weight models 1 and 2 are used more, as mentioned above. In these models the fatigue load is taken up by the top and shear reinforcement and there is not much tension in the bottom of the slab.

With increasing dimensions the self weight causes higher stress in the slab, while the fatigue load is unchanged. The entire fatigue load is concentrated to the section under the tower and when changing the width this section stays the same. When increasing the height however, it changes. A consequence of this change is that with increased height more fatigue load is carried vertically and less horizontally. This is seen in the figures that show the damage of the top and shear reinforcement for constant reinforcement amount and height or width of slab. For the varying width and constant reinforcement amount it is seen that the damage of the top and shear reinforcement is increasing with increasing width. For the varying heights and constant reinforcement amount the damage of the top reinforcement is decreasing with increasing height, while the damage of the shear reinforcement is increasing with increasing height.

The second fatigue assessment method for reinforcement, EC2:2, did not work well for the spectrum load used in this project. The reason for this is that the method checks if the fatigue life is sufficient, by comparing the capacity of the reinforcement with the highest stress caused by the fatigue load. It does not take the number of cycles into account, and this is crucial for this type of load since the maximum stress only comes with a number of cycles of 1000 in 20 years. This is also the reason why the reinforcement amounts estimated by the use of this method are significantly larger than the ones estimated by the damage calculation method. Figures 7.25-7.28 shows the reinforcement amounts estimated in the static analysis, as well as the required amounts for fatigue design according to EC2:1 and EC2:2.



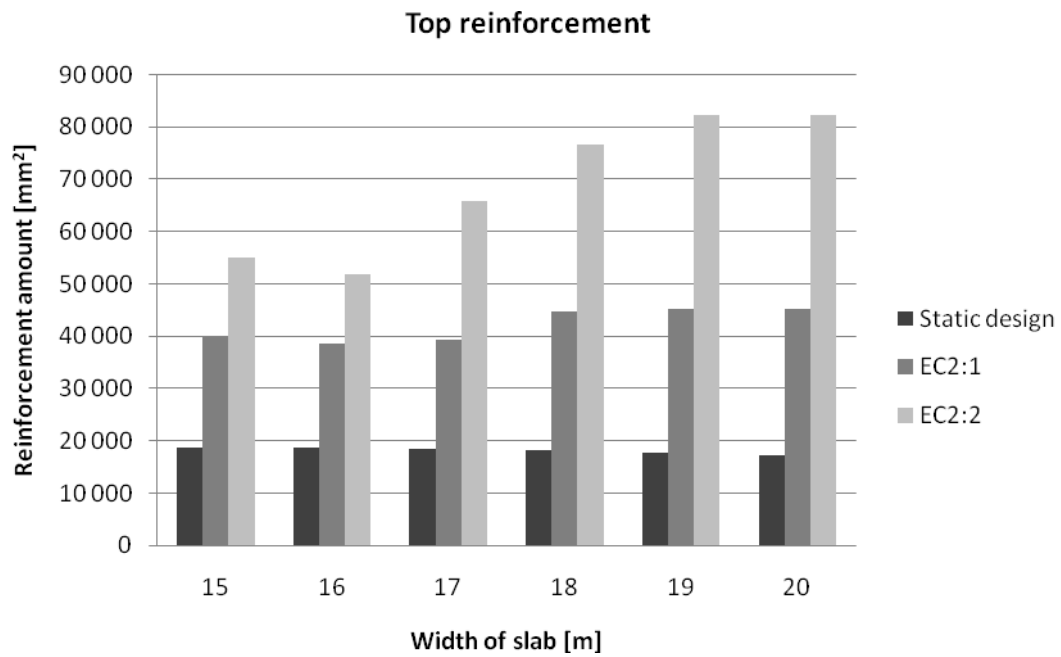


Figure 7.25 Top reinforcement amounts for varying widths for static design and fatigue design according to EC2:1 and EC2:2.

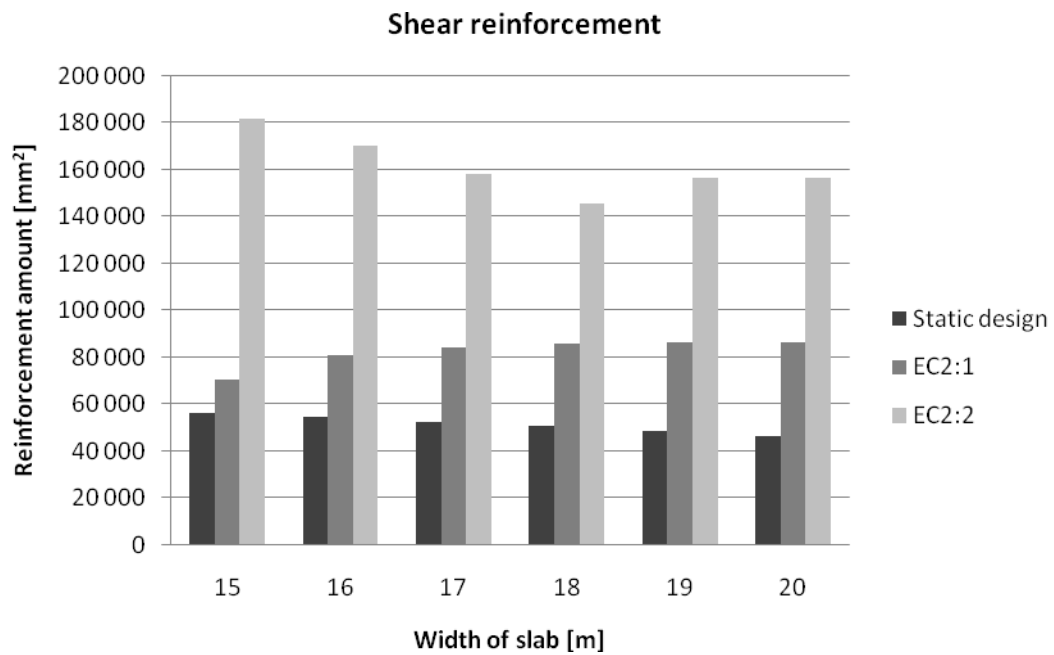


Figure 7.26 Shear reinforcement amounts for varying widths for static design and fatigue design according to EC2:1 and EC2:2.

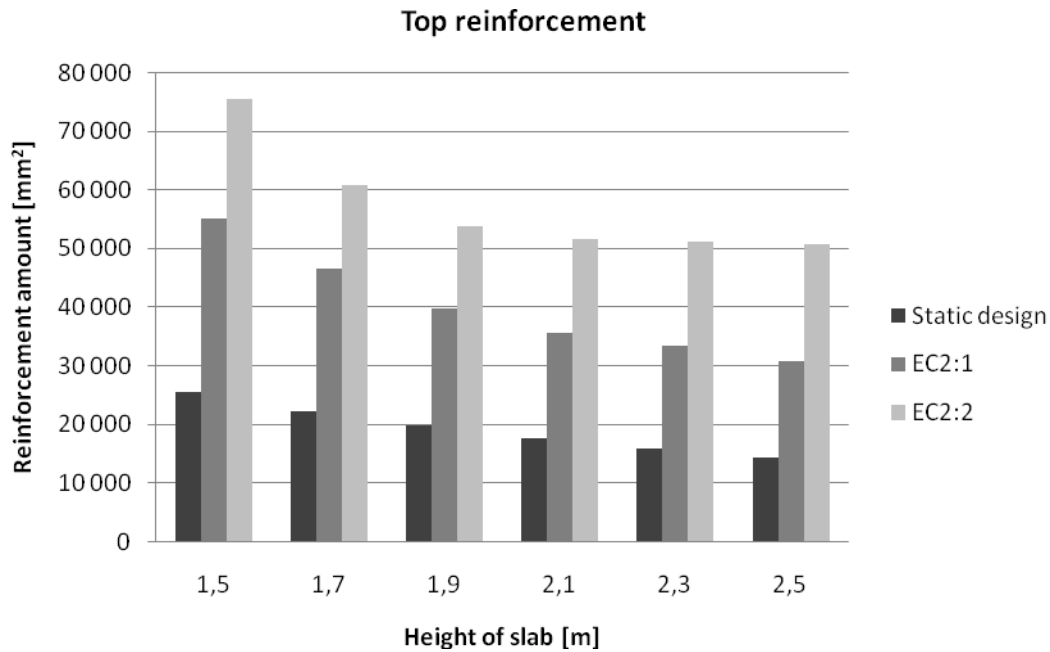


Figure 7.27 Top reinforcement amounts for varying heights for static design and fatigue design according to EC2:1 and EC2:2.

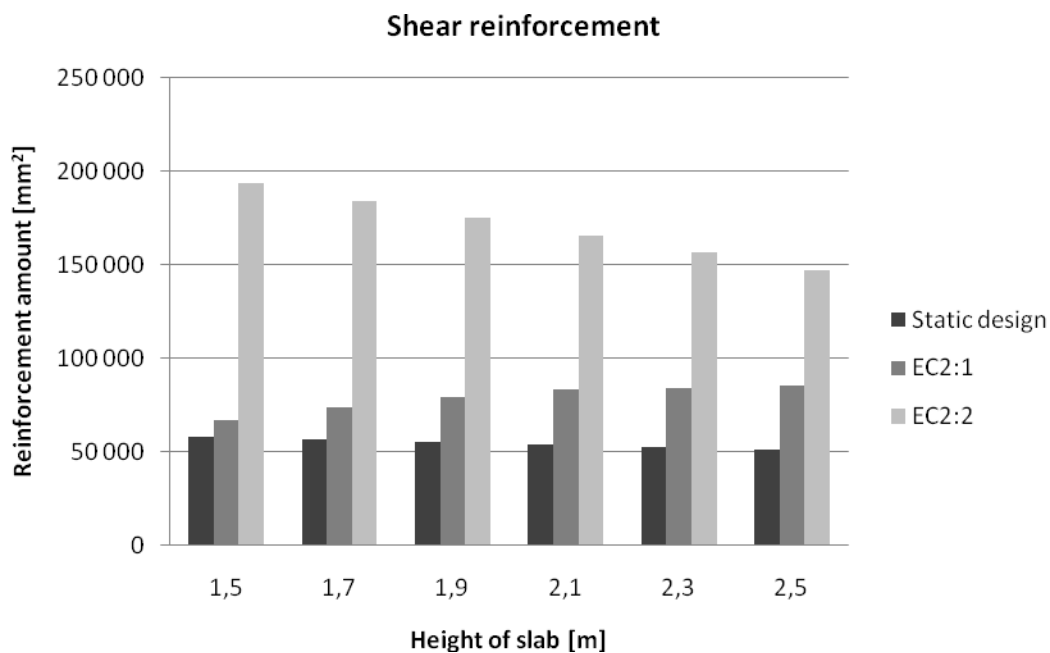


Figure 7.28 Shear reinforcement amounts for varying heights for static design and fatigue design according to EC2:1 and EC2:2.

## 7.4.2 Analysis of the results regarding concrete

The fatigue assessment of the concrete in compression showed very little or no damage for both damage calculation methods used in this project. The EC2 method gives a larger damage than the MC2010 method though. Even though the results in this parametric study showed very small differences in the results from the two

methods it could still be worth pointing out that there still are differences. It should also be noted that the first fatigue assessment method for compressed concrete, according to EC2, was rewritten according to EN 1992-2:2005. The original method that should be used for buildings did not work well for the spectrum load, for the same reason as one of the reinforcement methods. The second fatigue assessment method for concrete, according to EC2, does not work well for the spectrum load either since it does not take the frequency of the load into account. In this project however the stresses in the concrete are small enough, so this method gives a result showing a sufficient fatigue life of the concrete. If the damage calculation methods would show damage closer to the limit however, this method would imply an exceeded fatigue life.

In order to ensure a sufficient static design the node height, in the compression node under the tower, was increased. This is where fatigue of the concrete is checked since this is the most critical section for compression of the concrete. In the fatigue assessment the increased node height was used in the calculations, since a sufficient static design should be verified. This has naturally resulted in a lower damage of the concrete than what would have been if the initial node height had been used.

## 8 Conclusions and Recommendations

### 8.1 Fatigue design of foundation slabs

The aim of this project has been to give recommendations for design of foundation slabs with regard to fatigue. The static design of the foundation slab has however been proved to be more complex than what was first thought. This is due to the anchor ring that is used to anchor the tower to the slab. This ring will transfer the forces to the slab and will result in large sectional forces acting on a small area.

Both the concrete and the reinforcement in the slab are subjected to fatigue loading. In the static design of the slab some critical sections are identified. These sections are likely to be critical also for the fatigue assessment. In this project it is seen that the part of the slab that is affected by the fatigue load is the section inside the anchor ring. For the concrete the critical section is the compression node, at the top of the slab, under the tower.

A fatigue verification of the slab and a parametric study has been performed to investigate the behaviour of the different members of the slab, when it is subjected to fatigue loading. Three fatigue assessment methods regarding concrete have been used in the fatigue verification and the parametric study. Two of these methods are according to Eurocode 2 and one is according to fib Model Code 2010. Fatigue of the reinforcement was assessed with two fatigue assessment methods according to Eurocode 2. The parametric study has been performed both to find the parameters that have an impact on the fatigue life of the foundation slab and to compare the different fatigue assessment methods.

The parametric study showed that the dimensions of the slab have an impact on the fatigue life. With a larger slab the self weight increases, giving a lower damage to the bottom reinforcement. The work towards more optimized and minimized structures might result in that the fatigue load is governing for the bottom reinforcement.

An increasing height of slab resulted in more fatigue load being carried vertically and less horizontally, which gave higher fatigue of the shear reinforcement and less fatigue of the top reinforcement. An increasing width of slab resulted in decreasing fatigue of the bottom reinforcement and increasing fatigue of the top and shear reinforcement. The parametric study showed that fatigue is governing for the top and shear reinforcement for all the slab dimensions included in this study.

For fatigue design of a foundation slab for a wind power plant the method used for the fatigue assessment should be chosen so the whole spectrum of fatigue loads can be assessed. In this project it has been seen that the damage calculation methods are advantageous compared to the other methods. The methods for fatigue verification that do not take the frequency of the load into account are found to not be valid for such a complex fatigue load like the wind. In order to make these methods viable an equivalent value for the spectrum needs to be obtained.

It is seen when comparing the two fatigue assessment methods for the reinforcement that the needed amount of shear reinforcement according to the damage calculation method is increasing with increasing self weight of the slab. The results from EC2:2

show the opposite, a decreasing amount of shear reinforcement is needed for increasing self weight. This is due to that EC2:2 only checks the maximum stress range in the reinforcement and the maximum stress range in the shear reinforcement is decreasing with an increasing self weight. The total fatigue load taken up by the shear reinforcement is however increasing with an increased self weight, and this is seen in the results from the damage calculation method.

When calculating the damage of the concrete according to MC2010 and EC2:1 no significant damage was seen for either method. In order to compare the two methods very small damages were looked at and from these it could be concluded that the method according to MC2010 resulted in a lower damage than EC2:1.

Fatigue assessment regarding shear is in this project performed according to Eurocode 2. There are however uncertainties in the design code concerning how to perform the fatigue verification. The calculations regarding fatigue of the shear reinforcement are in this project performed according to the same methods as for the bottom and top reinforcement. The shear reinforcement is assumed to carry all of the shear force caused by the fatigue loading. The methods used could however be more accurate if the shear capacity of the concrete could contribute to the shear fatigue capacity.

## **8.2 Suggested future research**

To limit the project only one shape of slab has been investigated and the connections between the tower and the slab have not been checked for fatigue effects. One conclusion drawn was that the self weight of the slab has an impact on the fatigue damage of the members in the slab, while the shape of slab would be an interesting area to continue studying. Further, only gravity foundations have been studied and there are other types of foundations that may also be of interest with regard to fatigue, e.g. piled foundations.

In this project there have been uncertainties regarding the fatigue verification methods and which models to use. In the parametric study a connection between the fatigue damage and the choice of model could be seen, especially for fatigue loads close to limits between two different strut and tie models. To further investigate the force distribution in the slab, when subjected to fatigue loading, a FEM analysis could be done. The structural models used in this project are two dimensional; this is however a simplification and it would be more accurate to develop three dimensional models of the foundation slab.

The uncertainties regarding the fatigue verification methods are concerning fatigue assessment of the shear reinforcement. Further guidelines are needed to clarify the methods as well as improvements to make it possible to take the concrete capacity into consideration.

The speedy development of wind power plants with regard to efficiency is resulting in a short life span of the existing plants. In order to encourage a sustainable development in the construction industry a foundation slab could be used for several wind power plants. The life span of a wind power plant is today set to 20 years but a foundation slab, designed with regard to fatigue, can have a much longer life span.

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## A. Initial conditions for the slab

### A.1 Loads

The loads were calculated by Siemens according to the design code IEC 61400-1 Ed.3.  
The foundation loads are based on:

Annual average wind speed at hub height.....	$V_{ave}$ 8.5 m/s
10 min. extreme wind speed with return period of 50 years at hub height.....	$V_{ref}$ 42.5 m/s
3 sec. gust wind speed with return period of 50 years at hub height.....	$V_{50e}$ 59.5 m/s
Average air density.....	1.225 kg/m <sup>3</sup>

### Design sectional forces, ultimate limit state

Abnormal load case according to IEC 61400-1 Ed.3

Ultimate limit state

$F_Z := 3600 \cdot \text{kN}$	Normal Force
$F_{res} := 1080 \cdot \text{kN}$	Shear Force
$M_{res} := 97700 \cdot \text{kNm}$	Overturing Moment
$M_Z := 3800 \text{kNm}$	Torsional Moment

### Design sectional forces, serviceability limit state

Normal load case according to IEC 61400-1 Ed.3

Serviceability limit state

$F_{Z.SLS} := 3600 \cdot \text{kN}$	Normal Force
$F_{res.SLS} := 800 \cdot \text{kN}$	Shear Force
$M_{res.SLS} := 72500 \cdot \text{kNm}$	Overturing Moment
$M_{Z.SLS} := 7900 \cdot \text{kNm}$	Torsional Moment

### A.1 Material properties

#### Concrete

Assume a concrete strength class of C30/37.

Material properties according to [EN 1992-1-1:2005 3.1.3 table 3.1]

$f_{ck} := 30 \text{MPa}$	Characteristic compressive strength
$f_{cm} := 38 \text{MPa}$	Mean compressive strength at 28 days
$f_{ctm} := 2.9 \text{MPa}$	Mean tensile strength
$E_{cm} := 33 \text{GPa}$	Mean Young modulus for concrete
$\epsilon_{cu} := 3.5 \times 10^{-3}$	Ultimate strain

Design compressive strength

$$f_{cd} := \frac{f_{ck}}{\gamma_c} = 20 \cdot \text{MPa}$$

## Reinforcement

Assume reinforcement of type B500B

$$f_{yk} := 500 \text{MPa} \quad \text{Characteristic yield strength}$$

$$E_s := 200 \text{GPa} \quad \text{Young modulus for steel}$$

$$\gamma_s := 1.15 \quad \text{Partial coefficient according to [EN 1992-1-1:2005 2.4.2.4 table 2.1N]}$$

Design yield strength

$$f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \cdot \text{MPa}$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.061$$

## Post-tensioned anchor bolts

$$f_{p0.1k} := 900 \text{MPa}$$

$$\gamma_{ps} := 1.15 \quad \text{Partial coefficient for prestressing steel [EN 1992-1-1:2005 2.4.2.4 table 2.1N]}$$

$$f_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 782.609 \cdot \text{MPa}$$

## A.2 Geometry of slab

$$D := 16000 \text{mm} \quad \text{Length of slab}$$

$$h := 2000 \text{mm} \quad \text{Height of slab}$$

$$t := 750 \text{mm} \quad \text{The height over the slab which the horizontal force is acting on}$$

$$c_{con} := 50 \text{mm} \quad \text{Concrete cover}$$

$$\phi := 25 \text{mm} \quad \text{Diameter of the tensile reinforcement}$$

$$\phi_s := 12 \text{mm} \quad \text{Diameter of the shear reinforcement}$$

Depth to bottom reinforcement, placed in two directions and 2 layers.

$$d_1 := h - c_{con} - \phi_s - 0.5\phi = 1.925 \text{ m} \quad \text{Depth to the first reinforcement layer, unfavourable direction}$$

$$d_2 := h - c_{\text{con}} - \phi_s - 1.5 \cdot \phi = 1.9 \text{ m}$$

Depth to the second reinforcement layer, unfavourable direction

$$d_m := \frac{d_1 + d_2}{2} = 1.913 \text{ m}$$

Average depth to the bottom reinforcement

Depth to top reinforcement, placed in two directions and 2 layers

$$d'_1 := c_{\text{con}} + \phi_s + 0.5\phi = 0.075 \text{ m}$$

Depth to the first reinforcement layer, unfavourable direction

$$d'_2 := c_{\text{con}} + \phi_s + 1.5\phi = 0.1 \text{ m}$$

Depth to the second reinforcement layer, unfavourable direction

$$d'_m := \frac{d'_1 + d'_2}{2} = 0.087 \text{ m}$$

Average depth to the top reinforcement

Properties anchor ring (Siemens)

$$r_o := \frac{4200}{2} \text{ mm} = 2.1 \text{ m}$$

Outer radius

$$r_i := \frac{3560}{2} \text{ mm} = 1.78 \text{ m}$$

Inner radius

$$r_{\text{average}} := \frac{r_o + r_i}{2} = 1.94 \text{ m}$$

Average radius

### A.3 Self weight of the slab

Volume and unit weight

$$V_{\text{con}} := D^2 \cdot h = 512 \cdot \text{m}^3 \quad \text{Volume of concrete slab}$$

$$\gamma_{\text{con}} := 25 \frac{\text{kN}}{\text{m}^3} \quad \text{Unit weight of reinforced concrete}$$

$$\gamma_{\text{soil}} := 18 \frac{\text{kN}}{\text{m}^3} \quad \text{Unit weight of the soil}$$

Total self weight of the concrete

$$G_{\text{tot}} := V_{\text{con}} \cdot \gamma_{\text{con}} = 12.8 \cdot \text{MN}$$

Partial safety factors should be added to the self weight of the slab for ULS, SLS and EQU

#### ULS

Favourable

$$\gamma_G := 1.0 \quad G_{\text{ULS.fav}} := \gamma_G \cdot G_{\text{tot}} = 12.8 \cdot \text{MN}$$

Unfavourable

$$\gamma_{\text{G}} := 1.35 \quad G_{\text{ULS}} := \gamma_G \cdot G_{\text{tot}} = 17.28 \cdot \text{MN}$$

## SLS

$$G_{SLS} := G_{tot} = 12.8 \cdot \text{MN}$$

## EQU

Favourable

$$\gamma_{G.EQU} := 0.9 \quad G_{EQU.fav} := \gamma_{G.EQU} \cdot G_{tot} = 11.52 \cdot \text{MN}$$

Unfavourable

$$\gamma_{G.EQU} := 1.1 \quad G_{EQU} := \gamma_{G.EQU} \cdot G_{tot} = 14.08 \cdot \text{MN}$$

## **A.4 Soil pressure**

### Design sectional forces in ULS

$$F_Z = 3600 \cdot \text{kN} \quad F_{res} = 1080 \cdot \text{kN} \quad M_{res} = 97700 \cdot \text{kNm}$$

For the check of stability in ULS the self weight of the slab with partial factor for EQU is used [EN 1990/A1:2005]

$$R_{res} := F_Z + G_{EQU} = 17.68 \cdot \text{MN} \quad \text{Resulting vertical force}$$

The horizontal soil pressure at the depth of the slab, with the assumption that the ground water level is below the slab

$$k := 0.35 \quad \text{Coefficient depending on the type of soil}$$

$$p := k \cdot (\gamma_{soil} \cdot h) \cdot 1.5 = 18.9 \cdot \text{kPa} \quad \text{Multiply with 1.5 due to stiff construction}$$

$$P := \frac{1}{2} \cdot h \cdot p \cdot \frac{2 \cdot h}{3} = 25.2 \cdot \frac{\text{kNm}}{\text{m}} \quad \text{Horizontal soil pressure}$$

The horizontal soil pressure is favourable:

$$\gamma_{G.EQU} := 0.9$$

$$P_{EQU} := \gamma_{G.EQU} \cdot P = 22.68 \cdot \frac{\text{kNm}}{\text{m}}$$

Moment around the lower left corner gives the eccentricity of the resulting force from the soil

$$b := \frac{(F_Z + G_{EQU}) \cdot \frac{D}{2} - F_{res} \cdot (h + t) - M_{res} + P_{EQU} \cdot D}{R_{res}} = 2.327 \text{ m}$$

Compressed area

$$A_{com} := 2 \cdot b \cdot D = 74.449 \text{ m}^2$$

Vertical soil pressure, ULS

$$\sigma_{ULS} := \frac{R_{res}}{A_{com}} = 237.479 \cdot \text{kPa}$$

## B. Structural analysis

### B.1 Force distribution

A part of the anchor ring will be under compression due to the transfer of forces. To check how large part of the anchor ring that is under compression a simplified stress block approach is used. Assuming a ring with outer diameter  $r_o$  and inner diameter  $r_i$ . [EN 1992-1-1:2005 3.1.7 fig 3.5]

$$M_{Ed} := M_{res} + F_{res} \cdot t = 98510 \cdot \text{kNm}$$

$$\eta := 1.0 \quad \lambda := 0.8 \quad f_{ck} \leq 50 \text{MPa}$$

$$\eta \cdot f_{cd} \cdot b \cdot \lambda \cdot x_c \cdot \left( d - \beta \cdot x_c \right) - F_z \cdot \left( d - \frac{2 \cdot r_o}{2} \right) - M_{Ed} = 0$$

where  $\beta \cdot x_c$  is the distance from the compressed edge to the gravity centre of the simplified stress block and is calculated by:

$$\beta \cdot x_c = \frac{A_1 \cdot z_1 - A_2 \cdot z_2}{A_1 - A_2}$$

$A_1$  is the area of the whole circular segment, consisting of the anchor ring and the concrete area inside the ring.  $A_2$  is the area of the "inner" circular segment which is not included in the actual ring area.

$$k_1 = 2 \sqrt{\lambda \cdot x_c \cdot (2 \cdot r_o - \lambda \cdot x_c)}$$

$$k_2 = 2 \sqrt{\left[ \lambda \cdot x_c - (r_o - r_i) \right] \cdot \left[ 2 \cdot r_i - \left[ \lambda \cdot x_c - (r_o - r_i) \right] \right]}$$

$$s_1 = 2 \cdot r_o \cdot \text{asin} \left( \frac{k_1}{2 \cdot r_o} \right)$$

$$s_2 = 2 \cdot r_i \cdot \text{asin} \left( \frac{k_2}{2 \cdot r_i} \right)$$

Assume that a quarter of the anchor ring is in tension and that the anchor bolts are placed at the distance  $r_{average}$  from the centre of the anchor ring. The average depth from the top of the ring to the active anchor bolts then become:

$$d := r_o + \frac{r_{average} \cdot 2 \cdot \sin(45 \text{deg})}{2 \cdot 45 \text{deg}} = 3.847 \text{m}$$

Areas of circular segments 1 and 2.

$$A_1 = \frac{r_o \cdot (s_1 - k_1) + k_1 \cdot \lambda \cdot x_c}{2}$$

$$A_2 = \frac{r_i \cdot (s_2 - k_2) + k_2 \cdot \left[ \lambda \cdot x_c - (r_o - r_i) \right]}{2}$$

Distances to gravity centres for segment 1 and 2.

$$z_1 = r_o - \frac{k_1^3}{12 \cdot A_1}$$

$$z_2 = r_o - \frac{k_2^3}{12 \cdot A_2}$$

Given

$$x_c := 2.056 \text{ m}$$

$$\eta \cdot f_{cd} \cdot b \cdot \lambda \cdot x_c \cdot (d - \beta \cdot x_c) - F_z \cdot \left( d - \frac{2 \cdot r_o}{2} \right) - M_{Ed} = 0$$

$$x_c := \text{Find}(x_c) \quad x_c = 2.056 \text{ m}$$

Height of stress block:

$$h_c := \lambda \cdot x_c = 1.645 \text{ m}$$

Area of the entire anchor ring

$$A_{\text{ring}} := \frac{\pi}{4} \cdot \left[ (2r_o)^2 - (2r_i)^2 \right] = 3.901 \text{ m}^2$$

$$k_1 := 2 \sqrt{\lambda \cdot x_c \cdot (2 \cdot r_o - \lambda \cdot x_c)}$$

$$s_1 := 2 \cdot r_o \cdot \arcsin \left( \frac{k_1}{2 \cdot r_o} \right) = 5.68 \text{ m}$$

$$k_2 := 2 \sqrt{\left[ \lambda \cdot x_c - (r_o - r_i) \right] \cdot \left[ 2 \cdot r_i - \left[ \lambda \cdot x_c - (r_o - r_i) \right] \right]}$$

$$s_2 := 2 \cdot r_i \cdot \arcsin \left( \frac{k_2}{2 \cdot r_i} \right) = 4.672 \text{ m}$$

Areas of circular segments 1 and 2.

$$A_1 := \frac{r_o \cdot (s_1 - k_1) + k_1 \cdot h_c}{2} = 5.031 \text{ m}^2$$

$$A_2 := \frac{r_i \cdot (s_2 - k_2) + k_2 \cdot [h_c - (r_o - r_i)]}{2} = 3.375 \text{ m}^2$$

Distances to gravity centres for areas 1 and 2.

$$z_1 := r_o - \frac{k_1^3}{12 \cdot A_1} = 0.958 \text{ m}$$

$$z_2 := r_o - \frac{k_2^3}{12 \cdot A_2} = 1.093 \text{ m}$$

Area of effective stress block

$$A_{c0} := A_1 - A_2 = 1.656 \text{ m}^2$$

$$\frac{A_{c0}}{A_{\text{ring}}} = 42.463\% \quad \text{The part of the anchor ring that is compressed}$$

Distance to gravity centre of the compressed part of the anchor ring

$$z := \frac{A_1 \cdot z_1 - A_2 \cdot z_2}{A_1 - A_2} = 0.683 \text{ m}$$

## Moment and shear force diagram

The horizontal soil pressure has a small influence on the structural analysis, while this will be disregarded in the analysis.

In order to find the force distribution in the slab the sectional forces in ULS are replaced by a force couple. The distance between the compressive and tensile force acting on the slab is found from the calculations above:

$$l := d - z = 3.164 \text{ m}$$

Force couple  $F_C$  and  $F_T$

Compressive force transferred from the anchor ring to the slab

$$F_C := \frac{F_Z}{2} + \frac{(F_{\text{res}} \cdot t + M_{\text{res}})}{l} = 32.939 \cdot \text{MN}$$

Tensile force transferred from the anchor ring to the slab

$$F_T := -\frac{F_Z}{2} + \frac{(F_{\text{res}} \cdot t + M_{\text{res}})}{l} = 29.339 \cdot \text{MN}$$

**The self weight of the slab is favourable and will therefore be used with the partial safety factor for favourable loads in ULS.**

$$R_{\text{resv}} := F_Z + G_{\text{ULS.fav}} = 16.4 \cdot \text{MN}$$

$$b_{\text{ww}} := \frac{(F_z + G_{\text{ULS.fav}}) \cdot \frac{D}{2} - F_{\text{res}} \cdot t - M_{\text{res}}}{R_{\text{res}}} = 1.993 \text{ m}$$

Lengths of sections between forces

$$x := 0 \cdot \text{m}, 0.01 \cdot \text{m}.. D$$

$$b = 1.993 \text{ m} \quad \text{From the edge to the middle of the distributed reaction force}$$

$$b_2 := \frac{D}{2} - r_o + z = 6.583 \text{ m} \quad \text{From the edge to the first force of the force couple}$$

$$b_3 := \frac{D}{2} - r_o + d = 9.747 \text{ m} \quad \text{From the edge to the second force of the force couple}$$

Distributed forces

$$g_{\text{ww}} := \frac{G_{\text{ULS.fav}}}{D} = 0.8 \cdot \frac{\text{MN}}{\text{m}} \quad \text{Self weight}$$

$$q_s := \frac{R_{\text{res}}}{2 \cdot b} = 4.114 \cdot \frac{\text{MN}}{\text{m}} \quad \text{Reaction force}$$

Shear force distribution

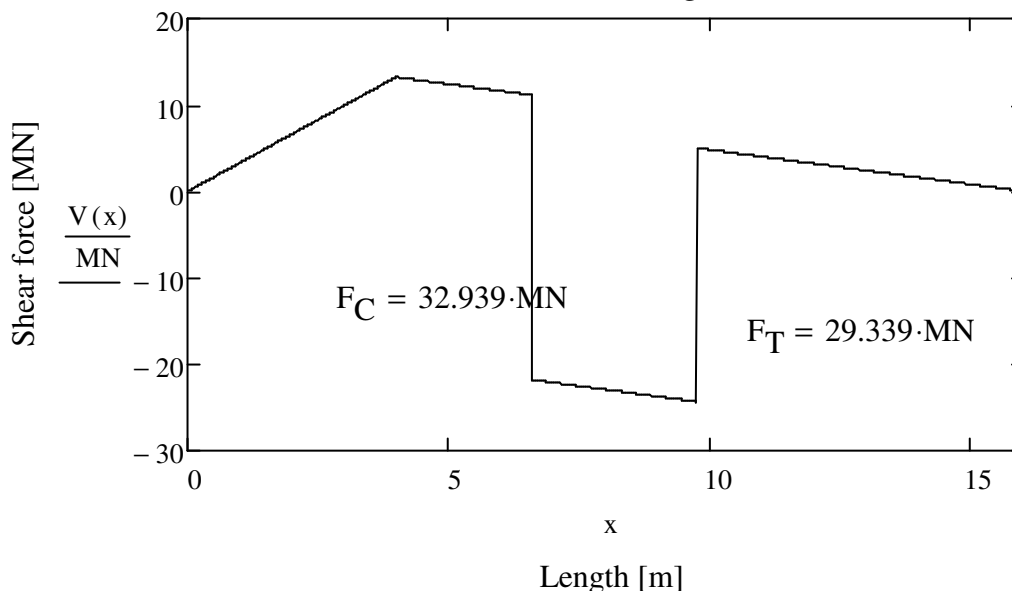
$$V(x) := \begin{cases} -g \cdot x + q_s \cdot x & \text{if } x \leq 2 \cdot b \\ -g \cdot x + q_s \cdot 2 \cdot b & \text{if } 2 \cdot b < x \leq b_2 \\ -g \cdot x + q_s \cdot 2 \cdot b - F_C & \text{if } b_2 < x \leq b_3 \\ -g \cdot x + q_s \cdot 2 \cdot b - F_C + F_T & \text{if } x > b_3 \end{cases}$$

$$V(2 \cdot b) = 13.211 \cdot \text{MN}$$

Maximum shear force

$$V(b_3) = -24.336 \cdot \text{MN}$$

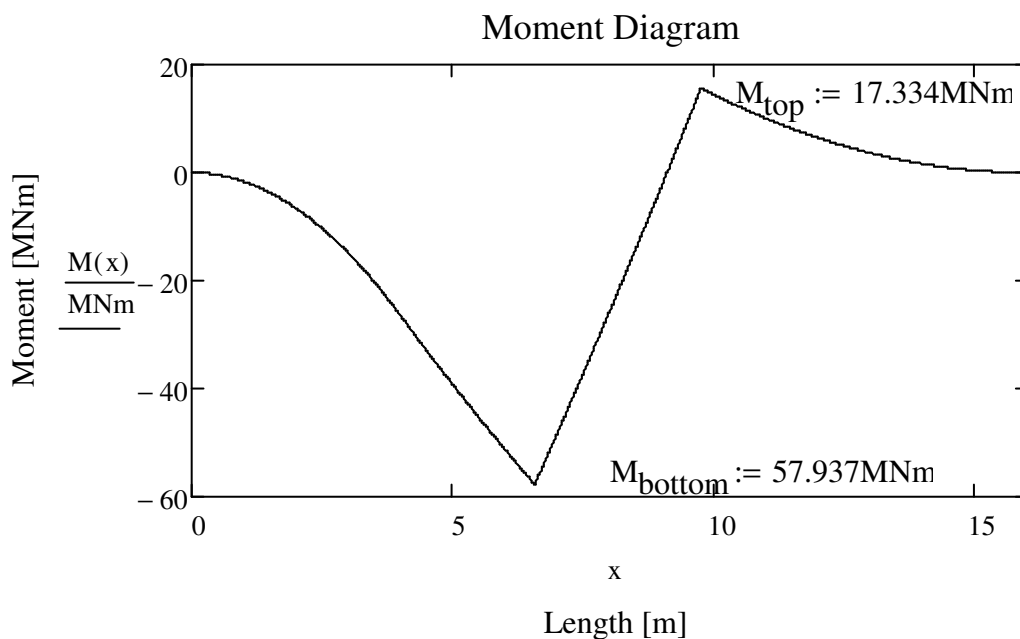
Shear force Diagram





Moment distribution

$$M(x) := \begin{cases} \frac{g \cdot x^2}{2} - \frac{q_s \cdot x^2}{2} & \text{if } x \leq 2 \cdot b \\ -q_s \cdot 2 \cdot b \cdot (x - b) + \frac{g \cdot x^2}{2} & \text{if } 2 \cdot b < x \leq b_2 \\ \frac{g \cdot (D - x)^2}{2} - F_T \cdot (b_3 - x) & \text{if } b_2 < x \leq b_3 \\ \frac{g \cdot (D - x)^2}{2} & \text{if } x > b_3 \end{cases}$$



## B.1 Ultimate limit state

### Bottom and top reinforcement

Amounts of bottom and top reinforcement are estimated in order to check the demand for minimum rotational stiffness.

$L_{\text{critical}} := b_2 = 6.583 \text{ m}$       The critical section is obtained from the moment diagram and is counted from the edge of the slab.

Moment that needs to be resisted by the bottom reinforcement

$$M_{\text{bottom}} := R_{\text{res}} \cdot (L_{\text{critical}} - b) - \frac{\frac{G_{\text{ULS.fav}}}{D} \cdot L_{\text{critical}}^2}{2} = 57937.059 \cdot \text{kNm}$$

Moment that needs to be resisted by the top reinforcement

$$M_{top} := \frac{\frac{G_{ULS.fav}}{D} \cdot L_{critical}^2}{2} = 17334.457 \cdot \text{kNm}$$

Bottom reinforcement amount

$$A_s := \frac{M_{bottom}}{f_{yd} \cdot (d_2 - d'_2)} = 73989.581 \cdot \text{mm}^2$$

Top reinforcement amount

$$A'_s := \frac{M_{top}}{f_{yd} \cdot (d_2 - d'_2)} = 22137.286 \cdot \text{mm}^2$$

## Rotational stiffness

Required combined minimum rotational stiffness around horizontal axis of soil and foundation:  
1500 MNm/deg

Rotational stiffness is checked in ULS by assuming a cantilever. The reaction force from the ground is a uniformly distributed load over the length  $2b$ . The self weight of the slab is counteracting the rotation.

Given

$$x_{II} := 0.295 \text{ m}$$

$$D \cdot \frac{x_{II}^2}{2} + (\alpha - 1) \cdot A'_s \cdot (x_{II} - d'_2) - \alpha \cdot A_s \cdot (d_2 - x_{II}) = 0$$

$$x_{II} := \text{Find}(x_{II}) \quad x_{II} = 0.295 \text{ m}$$

$$I_{II} := \frac{D \cdot x_{II}^3}{3} + (\alpha - 1) \cdot A'_s \cdot (x_{II} - d'_2)^2 + \alpha \cdot A_s \cdot (d_2 - x_{II})^2 = 1.297 \text{ m}^4$$

$$M_{tot} := F_{res} \cdot \left( \frac{h}{2} + t \right) + M_{res} = 99.59 \cdot \text{MNm}$$

$$E_{cm} = 33 \cdot \text{GPa}$$

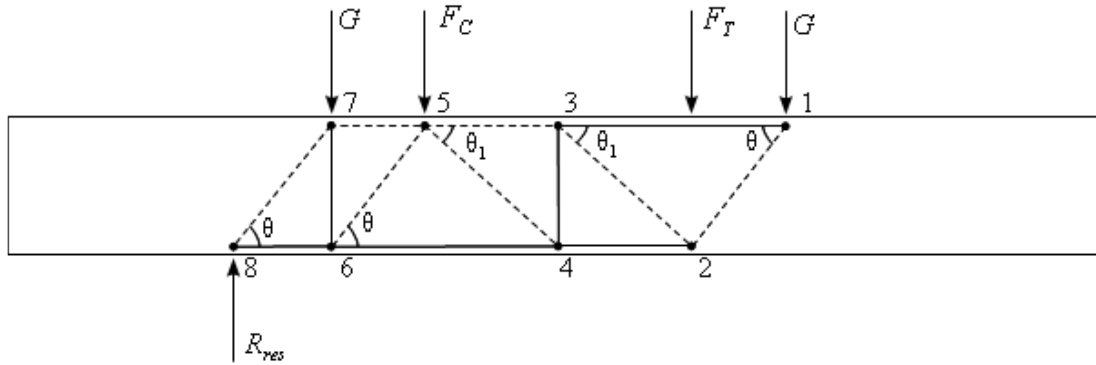
$$a := \frac{D}{2} - 2 \cdot b = 4.013 \text{ m}$$

$$\theta := \frac{q_s \cdot (2 \cdot b)^3}{6 \cdot E_{cm} \cdot I_{II}} + \frac{q_s \cdot (2 \cdot b)^2 \cdot a}{4 \cdot E_{cm} \cdot I_{II}} + \frac{q_s \cdot (2 \cdot b) \cdot \left( \frac{D}{2} - b \right) \cdot a}{2 \cdot E_{cm} \cdot I_{II}} - \frac{g \cdot \left( \frac{D}{2} \right)^3}{6 \cdot E_{cm} \cdot I_{II}} = 0.319 \cdot \text{deg}$$

$$\text{rotational stiffness} := \frac{M_{\text{tot}}}{\theta \cdot \text{deg}} = 17876.701 \cdot \frac{\text{MNm}}{\text{deg}} \quad \text{OK}$$

### Strut and tie model and design of compression - compression node

A strut and tie model is established to find the required reinforcement amounts. Under the compressive force from the tower there is a risk of crushing of the concrete. Therefore a check of the compressive stresses in node 5 needs to be done.



Node height:

$$u_{\text{node}} := 2 \cdot (h - d_m) = 0.174 \text{ m}$$

The node height is increased to ensure that the concrete in node 5 has sufficient compressive strength.

$$b_{\text{w}} := \frac{(F_Z + G_{\text{ULS.fav}}) \cdot \frac{D}{2} - F_{\text{res}} \cdot (h + t) - M_{\text{res}}}{R_{\text{res}}} = 1.862 \text{ m}$$

$$\theta_{\text{w}} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left( \frac{\frac{D}{2} - \frac{1}{2} - b}{2} \right)} \right] = 38.326 \cdot \text{deg}$$

$$\theta_1 := \text{atan} \left[ \frac{(d_2 - d'_2)}{\frac{1}{2}} \right] = 48.708 \cdot \text{deg}$$

$$F_{\text{T}} := \frac{F_Z}{2} - \frac{F_{\text{res}} \cdot (h + t) + M_{\text{res}}}{1} = -30.021 \cdot \text{MN}$$

$$F_{\text{C}} := \frac{F_Z}{2} + \frac{F_{\text{res}} \cdot (h + t) + M_{\text{res}}}{1} = 33.621 \cdot \text{MN}$$

$$G := \frac{G_{\text{ULS.fav}}}{2} = 6.4 \cdot \text{MN}$$

Node 1

$$C_{1.2} := \frac{G}{\sin(\theta)} = 10.32 \cdot \text{MN}$$

$$T_{1.3} := \cos(\theta) \cdot C_{1.2} = 8.096 \cdot \text{MN}$$

Node 2

$$F_T = -30.021 \cdot \text{MN}$$

$$C_{2.3} := \frac{-F_T}{\sin(\theta_1)} - \frac{\sin(\theta) \cdot C_{1.2}}{\sin(\theta_1)} = 31.439 \cdot \text{MN}$$

$$T_{2.4} := \cos(\theta_1) \cdot C_{2.3} - \cos(\theta) C_{1.2} = 12.65 \cdot \text{MN}$$

Node 3

$$C_{3.5} := \cos(\theta_1) \cdot C_{2.3} - T_{1.3} = 12.65 \cdot \text{MN}$$

$$T_{3.4} := \sin(\theta_1) \cdot C_{2.3} = 23.621 \cdot \text{MN}$$

Node 4

$$C_{4.5} := \frac{T_{3.4}}{\sin(\theta_1)} = 31.439 \cdot \text{MN}$$

$$T_{4.6} := T_{2.4} + \cos(\theta_1) \cdot C_{4.5} = 33.397 \cdot \text{MN}$$

Node 5

$$F_C = 33.621 \cdot \text{MN}$$

$$C_{5.6} := \frac{F_C - \sin(\theta_1) \cdot C_{4.5}}{\sin(\theta)} = 16.125 \cdot \text{MN}$$

$$C_{5.7} := C_{3.5} + \cos(\theta_1) \cdot C_{4.5} - \cos(\theta) \cdot C_{5.6} = 20.746 \cdot \text{MN}$$

Node 6

$$T_{6.7} := \sin(\theta) \cdot C_{5.6} = 10 \cdot \text{MN}$$

$$T_{6.8} := T_{4.6} - \cos(\theta) \cdot C_{5.6} = 20.746 \cdot \text{MN}$$

Node 7

$$C_{7.8} := \frac{C_{5.7}}{\cos(\theta)} = 26.446 \cdot \text{MN}$$

$$T_{6.7} = \sin(\theta) \cdot C_{7.8} - G$$

Node 8

$$T_{6.8} = \cos(\theta) \cdot C_{7.8} = 1$$

$$R_{\text{res}} = \sin(\theta) \cdot C_{7.8}$$

Compressive stress under the compressive force  $F_C$

$$C_{c0.5} := C_{3.5} + \cos(\theta_1) \cdot C_{4.5} = 33.397 \cdot \text{MN}$$

$$k_{\text{average}} := \frac{k_1 + k_2}{2} = 3.771 \text{ m}$$

$$A_{c0.5} := u_{\text{node}} \cdot k_{\text{average}} = 0.656 \text{ m}^2$$

$$\sigma_{c0.5.\text{static}} := \frac{C_{c0.5}}{A_{c0.5}} = 50.898 \cdot \text{MPa}$$

The maximum allowable stress in the node is:

[EN 1992-1-1:2005 6.5.4 (6)]

$$k_4 := 3 \quad \text{National parameter}$$

$$v := 1 - \frac{f_{ck}}{250 \text{ MPa}} = 0.88 \quad \text{National parameter}$$

$$f_{cd.c} := k_4 \cdot v \cdot f_{cd} = 52.8 \cdot \text{MPa}$$

$$\sigma_{Rd.\text{max}} := f_{cd.c} = 52.8 \cdot \text{MPa}$$

$$\sigma_{c0.5.\text{static}} \leq \sigma_{Rd.\text{max}} = 1$$

### Amount of bottom reinforcement

$$T_1 := \max(T_{2.4}, T_{4.6}, T_{6.8}) = 33.397 \cdot \text{MN} \quad \text{Maximum force in bottom reinforcement}$$

$$A_s := \frac{T_1}{f_{yd}} = 76812.274 \cdot \text{mm}^2$$

$$\phi = 25 \cdot \text{mm} \quad \text{assumed diameter of the reinforcement bar.}$$

$$n := \frac{A_s \cdot 4}{\pi \cdot \phi^2} = 156.481 \quad n := 157 \quad \text{Number of bars required}$$

$$cc := \frac{2(D - n \cdot \phi - 2c_{\text{con}})}{(n - 1)} = 153.526 \cdot \text{mm} \quad \text{Maximum spacing between bars}$$

$$cc := 150 \text{ mm}$$

### Amount of top reinforcement

$$T_2 := T_{1.3} = 8.096 \cdot \text{MN}$$

There is only one tie in the top of the slab

$$A'_{\text{ss}} := \frac{T_2}{f_{yd}} = 18621.157 \cdot \text{mm}^2$$

### Amount of shear reinforcement outside insert ring

$$T_{6.7} = 10 \cdot \text{MN}$$

$$A_{\text{ss.out}} := \frac{T_{6.7}}{f_{yd}} = 23000 \cdot \text{mm}^2$$

### Amount of shear reinforcement inside insert ring

$$T_{3.4} = 23.621 \cdot \text{MN}$$

$$A_{\text{ss.in}} := \frac{T_{3.4}}{f_{yd}} = 54329.218 \cdot \text{mm}^2$$

The differences between the reinforcement amounts estimated earlier and the ones from the strut and tie model depends for example on where the moment equilibrium is calculated. In the strut and tie model a total equilibrium is assumed and the moment equilibrium equation is therefore calculated for the horizontal force acting on the height of the slab plus the height of the anchor ring. For the reinforcement amounts estimated earlier the moment equilibrium is estimated in the interface between the tower and the slab. The tensile force  $F_T$  and the compressive force  $F_C$  are acting at a distance  $l$  from each other. In the strut and tie model the nodes these forces are acting in are placed at a distance  $l/2$  from the middle of the slab. In reality the distance from the middle of the slab to the compressive force is smaller than the distance from the middle of the slab to the tensile force. In the strut and tie model the self weight of the slab is modelled as 2 concentrated loads, while in the force distribution above it is modelled as a uniformly distributed load. These are all reasons for the differences between the calculated reinforcement amounts.

### Shear force

[EN 1992-1-1:2005 6.2.2]

The worst case for checking the shear capacity is using the effective depth for the second layer of reinforcement,  $d_2$

$$V_{\text{Ed}} := R_{\text{res}} - 2 \cdot b \cdot g = 13.421 \cdot \text{MN} \quad \text{Critical shear force}$$

$$V_{\text{Rd.c}} = C_{\text{Rd.c}} \cdot k \cdot \left( 100 \rho_2 \cdot f_{\text{ck}} \right)^{\frac{1}{3}} \cdot D \cdot d \geq v_{\text{min}} \cdot D \cdot d$$

$$v_{\text{min}} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{\text{ck}}^{\frac{1}{2}}$$

$$C_{Rd.c} := \frac{0.18}{\gamma_c} = 0.12$$

$$k := 1 + \sqrt{\frac{200\text{mm}}{d_2}} = 1.324 \quad k \leq 2.0 = 1$$

$$A_{s1} := A_s = 76812.274 \cdot \text{mm}^2$$

$$\rho_2 := \frac{A_{s1}}{D \cdot d_2} = 0.253 \cdot \%$$

$$V_{Rd.c} := C_{Rd.c} \cdot \text{MPa} \cdot k \cdot \left( 100 \cdot \rho_2 \cdot \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{3}} \cdot D \cdot d_2 = 9.492 \cdot \text{MN}$$

$$V_{Rd.c} \geq 0.035 \cdot \text{MPa} \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{2}} \cdot D \cdot d_2 = 1$$

$$V_{Ed} \leq V_{Rd.c} = 0 \quad \text{Shear reinforcement required!}$$

Amount of shear reinforcement. The shear reinforcement resists all of the critical shear force.

$$A_{ss} := \frac{V_{Ed}}{f_{yd}} = 30869.366 \cdot \text{mm}^2$$

Check of how far out in the slab the shear reinforcement has to be placed. Shear reinforcement is needed in every section where the shear force is higher than the shear capacity,  $V_{Rd.c}$ . From the shear force distribution it is seen that this section will be within the distance  $2b$  from the edge of the slab.

Given

$$x := 2.865\text{m}$$

$$-g \cdot x + q_s \cdot x = V_{Rd.c}$$

$$x := \text{Find}(x) \quad x = 2.865\text{m}$$

Choose to place the shear reinforcement out to a distance of 2.8 meters from the edge of the slab. In this section the shear force is checked to be lower than the shear capacity.

$$x := 2.8\text{m}$$

$$V_x := -g \cdot x + q_s \cdot x$$

$$V_x \leq V_{Rd.c} = 1 \quad \text{OK!}$$

## Punching Shear

[EN 1992-1-1:2005 6.4]

The anchor ring transfers forces from the tower to the concrete slab. The lower part of the anchor ring, that is embedded in the concrete slab, is the anchorage of the tensile forces from the tower. The compressive forces from the tower can be assumed to be transferred directly from the upper part of the anchor ring, which is not embedded in the concrete slab, to the top of the foundation slab. Due to the compressive forces punching shear needs to be checked. This check is done by assuming punching shear under a circular column.

$i := 0..3$

$$d_m = 1.913 \text{ m} \quad d_{ef} := d_m - \frac{\phi}{2} = 1.9 \text{ m} \quad \text{Effective depth of the slab}$$

$$v_{Rd,c} = C_{Rd,c} \cdot k \cdot \left( 100 \rho_l f_{ck} \right)^{\frac{1}{3}} \geq v_{min}$$

$$\theta_i := \begin{pmatrix} 40 \\ 35 \\ 30 \\ 26 \end{pmatrix} \text{ deg} \quad \text{Different angles that are checked with regard to punching to find the critical section.}$$

$$d_i := \frac{d_{ef}}{\tan(\theta_i)} = \dots \quad d = \begin{pmatrix} 2.265 \\ 2.714 \\ 3.292 \\ 3.897 \end{pmatrix} \text{ m} \quad \text{Distance to the control section}$$

$$r_{cont} := r_o + d = \begin{pmatrix} 4.365 \\ 4.814 \\ 5.392 \\ 5.997 \end{pmatrix} \text{ m} \quad \text{Radius of the control section}$$

$$u := 2 \cdot \pi \cdot r_{cont} = \begin{pmatrix} 27.426 \\ 30.248 \\ 33.877 \\ 37.678 \end{pmatrix} \text{ m} \quad \text{The length of the control section is the perimeter of the circle with the radius } r_{cont}$$

$$k = 1.324$$

$$C_{Rd,c} = 0.12 \quad \text{Same as those calculated for shear capacity } V_{Rd,c}$$

$$\rho_{2v} := \frac{A_{s1}}{D \cdot d_2} \quad \rho_1 := \frac{A_{s1}}{D \cdot d_1} \quad \rho := \sqrt{\rho_1 \rho_2} = 0.002509603$$



Bearing capacity for punching shear

$$v_{Rd.c} := C_{Rd.c} \cdot \text{MPa} \cdot k \cdot \left( 100 \cdot \rho \cdot \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{3}} = 0.311 \cdot \text{MPa}$$

$$v_{Rd.c} \geq 0.035 \cdot \text{MPa} \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{2}} = 1$$

Eccentricity

$$e_{ecc} := \frac{M_{Ed}}{V_{Ed}} = 7.34 \text{ m} \quad \text{M}_{Ed} := M_{res} + F_{res} \cdot t = 98.51 \cdot \text{MNm} \quad \text{V}_{Ed} := F_z = 3.6 \cdot \text{MN}$$

$$\beta_i := 1 + 0.6\pi \cdot \frac{e_{ecc}}{2 \cdot r_o + 4 \cdot d_i} = \dots \quad \text{For circular inner columns}$$

$$\sigma_j := \frac{F_z}{2 \cdot bD} = 60.432 \cdot \text{kPa}$$

Load inside the control perimeter, reduced with the self weight of the slab.

$$\Delta V_{Ed_i} := \begin{cases} 0 & \text{if } r_{cont_i} < \frac{D}{2} - 2 \cdot b \\ \left[ \frac{4}{3} \sqrt{\left( r_{cont_i} \right)^2 - \left( \frac{D}{2} - 2 \cdot b \right)^2} \left[ r_{cont_i} - \left( \frac{D}{2} - 2 \cdot b \right) \right] \right] \cdot \sigma_j & \text{otherwise} \end{cases} = \dots \cdot \text{MN}$$

Shear force at the control perimeter, reduced with the compressive force from the ground.

$$V_{Ed.red_i} := V_{Ed} - \Delta V_{Ed_i} = \dots \cdot \text{MN}$$

Maximum shear stress

$$v_{Ed_i} := \beta_i \cdot \frac{V_{Ed.red_i}}{u_i \cdot d_{ef}} = \dots \cdot \text{MPa} \quad v_{Ed} = \begin{pmatrix} 0.141 \\ 0.117 \\ 0.092 \\ 0.072 \end{pmatrix} \cdot \text{MPa}$$

$$v_{Ed} \leq v_{Rd.c} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{OK!}$$

## Minimum and maximum reinforcement amount

### Ultimate limit state

[EN 1992-1-1:2005 9.3.1.1]

$b_t := D$  Average width of the tensile zone of the section.

### Minimum reinforcement area ULS

$$A_{s,min,ULS} := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_t \cdot d_m = 46156.864 \cdot \text{mm}^2 \quad [\text{EN 1992-1-1:2005 9.2.1.1 (1)}]$$

$$A_{s,min,ULS} > 0.0013 \cdot b_t \cdot d_m = 1 \quad \text{OK!}$$

Check that the minimum reinforcement amount is smaller than the estimated reinforcement area.

$$A_s > A_{s,min,ULS} = 1 \quad \text{OK!}$$

### Maximum reinforcement area ULS

Area of the concrete cross section

$$A_c := D \cdot h = 32 \text{ m}^2$$

Maximum reinforcement area ULS

$$A_{s,max} := 0.04 \cdot A_c = 1.28 \text{ m}^2$$

Check that the maximum reinforcement amount is larger than the estimated reinforcement area.

$$A_s < A_{s,max} = 1 \quad \text{OK!}$$

According to EN 1992-1-1:2005 9.3.1.1 the maximum distance between reinforcement bars should not be larger than:

$$s_{\max,slabs} := \begin{cases} 3 \cdot h & \text{if } 3 \cdot h \leq 400 \text{ mm} \\ 400 \cdot \text{mm} & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{for main reinforcement} \\ \text{where } h \text{ is the total height of the slab} \end{array}$$

$$cc \leq s_{\max,slabs} = 1 \quad \text{OK!}$$

## Post-tensioned anchor bolts

The properties for the anchor bolts are provided by the supplier of the wind power plant and are in this project M42 bolts with a pretressing force of 400 kN. The number of bolts is 100 in both the inner and outer bolt circle.

$\phi_b := 42\text{mm}$  Area of one anchor bolt

$$A_p := \pi \cdot \left( \frac{\phi_b}{2} \right)^2 = 1385.442 \cdot \text{mm}^2$$

Design capacity of one anchor bolt

$$F := f_{pd} \cdot A_p = 1084.259 \cdot \text{kN}$$

As assumed before one quarter of the anchor ring is in tension. This gives that the number of active bolts is:

$$n_p := 2 \left( \frac{100}{4} \right) = 50$$

With 50 active anchor bolts the capacity is:

$$F_{\text{tot}} := F \cdot n_p = 54.213 \cdot \text{MN}$$

$$F_{\text{tot}} > |F_T| = 1 \quad \text{OK!}$$

$$n_b := \frac{|F_T|}{F} = 27.688 \quad \text{The needed amount of active anchor bolts to anchor the tensile force.}$$

## B.2 Serviceability limit state

### Crack width

[EN 1992-1-1:2005 7.3.4]

Steel stress in SLS state II

In serviceability limit state the self weight of the slab is used with a partial factor for SLS. The horizontal soil pressure is disregarded.

$$R_{\text{res.SLS}} := F_{Z.\text{SLS}} + G_{\text{SLS}} = 16.4 \cdot \text{MN}$$

$$b_{\text{SLS}} := \frac{(F_{Z.\text{SLS}} + G_{\text{SLS}}) \cdot \frac{D}{2} - F_{\text{res.SLS}} \cdot (h + t) - M_{\text{res.SLS}}}{R_{\text{res.SLS}}} = 3.445 \text{ m}$$

$$M_{\text{bottom.SLS}} := R_{\text{res.SLS}} \cdot (L_{\text{critical}} - b_{\text{SLS}}) - \frac{G_{\text{SLS}}}{D} \cdot L_{\text{critical}}^2 = 34127.059 \cdot \text{kNm}$$

Given

$$x_{\text{II}} := 0.301 \text{ m}$$

$$D \cdot \frac{x_{\text{II}}^2}{2} + (\alpha - 1) \cdot A'_s \cdot (x_{\text{II}} - d'_2) - \alpha \cdot A_s \cdot (d_2 - x_{\text{II}}) = 0$$

$$x_{II} := \text{Find}(x_{II}) \quad x_{II} = 0.301 \text{ m}$$

$$I_{II} := \frac{D \cdot x_{II}^3}{3} + (\alpha - 1) \cdot A'_s \cdot (x_{II} - d'_2)^2 + \alpha \cdot A_s \cdot (d_2 - x_{II})^2 = 1.34 \text{ m}^4$$

$$\sigma_s := \alpha \cdot \frac{M_{\text{bottom.SLS}}}{I_{II}} \cdot (d_2 - x_{II}) = 246.81 \cdot \text{MPa}$$

$$w_k = s_{r.\text{max}}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ct.\text{eff}}}{\rho_{p.\text{eff}}} \cdot (1 + \alpha_e \cdot \rho_{p.\text{eff}})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

Maximum allowed crack width is dependent on life time and exposure class etc.  
(L50 and XC2)

$$w_{k.\text{max}} := 0.45 \text{ mm} \quad \text{Maximum allowed crack width according to [EN 1992-1-1:2005 NA]}$$

$$k_t := 0.4 \quad \text{Long term duration of load}$$

$$f_{ct.\text{eff}} := f_{ctm} = 2.9 \cdot \text{MPa}$$

$$A_{c.\text{eff}} := \min \left[ 2.5 \cdot (h - d_m), \frac{(h - x_{II})}{3}, \frac{h}{2} \right] \cdot D = 3.48 \text{ m}^2 \quad \text{Effective area}$$

$$\rho_{p.\text{eff}} := \frac{A_s}{A_{c.\text{eff}}} = 0.022$$

$$E_s = 200 \cdot \text{GPa}$$

$$\alpha_e := \frac{E_s}{E_{cm}} = 6.061$$

$$\sigma_s = 246.81 \cdot \text{MPa} \quad \text{Steel stress}$$

$$\Delta \varepsilon = \varepsilon_{sm} - \varepsilon_{cm}$$

$$\Delta \varepsilon := \frac{\sigma_s - k_t \cdot \frac{f_{ct.\text{eff}}}{\rho_{p.\text{eff}}} \cdot (1 + \alpha_e \cdot \rho_{p.\text{eff}})}{E_s} = 0.001$$

$$\Delta \varepsilon := \begin{cases} 0.6 \frac{\sigma_s}{E_s} & \text{if } \Delta \varepsilon \leq 0.6 \frac{\sigma_s}{E_s} \\ \Delta \varepsilon & \text{otherwise} \end{cases} = 0.001$$

$$s_{r,max} = k_3 \cdot c_{con} + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p,eff}}$$

$$\phi = 25 \cdot \text{mm} \quad \text{Diameter of the reinforcement}$$

$$c_{con} = 50 \cdot \text{mm} \quad \text{Concrete cover}$$

$$k_1 := 0.8 \quad \text{Good interaction between concrete and reinforcement}$$

$$k_2 := 1.0 \quad \text{No prestressed reinforcement}$$

$$k_3 := 7 \cdot \frac{\phi}{c_{con}} = 3.5 \quad \text{According to national annex}$$

$$k_4 := 0.245 \quad \text{According to national annex}$$

$$s_{r,max} := k_3 \cdot c_{con} + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p,eff}} = 0.397 \text{ m}$$

$$w_k := s_{r,max} \cdot \Delta \epsilon = 0.372 \cdot \text{mm} \quad \text{Crack width}$$

$$w_k \leq w_{k,max} = 1 \quad \text{OK!} \quad \text{Check if estimated crack width is smaller than maximum allowed crack width according to national annex in EN 1992-1-1:2005}$$

## Minimum reinforcement amount for limitation of crack width

### Serviceability limit state

[EN 1992-1-1:2005 7.3.2]

$$f_{yk} = 500 \cdot \text{MPa}$$

$$k_c := 1.0 \quad \text{Pure tension}$$

$$k_s := 0.65 \quad h \geq 800 \text{ mm}$$

$$f_{ct,eff} = 2.9 \cdot \text{MPa} \quad \text{Average concrete tensile strength at the time when the first crack is expected. Assume this will occur after 28 days.}$$

$$\alpha := \frac{E_s}{E_{cm}} = 6.061$$

$$A_I := A_c + (\alpha - 1) \cdot A'_s + (\alpha - 1) \cdot A_s = 32.483 \text{ m}^2$$

$$x_I := \frac{A_c \cdot \frac{h}{2} + (\alpha - 1) \cdot A'_s \cdot d'_m + (\alpha - 1) \cdot A_s \cdot d_m}{A_I} = 1.008 \text{ m}$$

$$A_{ct} := D \cdot (h - x_I) = 15.868 \cdot m^2 \quad \text{Area of the tensile zone before the first crack}$$

Minimum reinforcement area SLS

$$A_{s,min} := \frac{k_c \cdot k \cdot f_{ct,eff} A_{ct}}{f_{yk}} = 0.06 m^2$$

Check that the minimum reinforcement amount is smaller than the estimated reinforcement area.

$$A_s \geq A_{s,min} = 1 \quad \text{OK!} \quad A_s = 0.077 m^2$$


## C. Fatigue assessment background calculations

### C.1 Fatigue load input


Fatigue moment and horizontal force imported from excel.


$M_{\text{Maxover}} :=$   Maximum overturning moment in a cycle.

$M_{\text{Minover}} :=$   Minimum overturning moment in a cycle

$n_{\text{over}} :=$   Number of cycles overturning moment

$F_{\text{Fatmax}} :=$   Maximum horizontal force in a cycle

$F_{\text{Fatmin}} :=$   Minimum horizontal force in a cycle

$n_F :=$   Number of cycles horizontal force

$M_{\text{fat.max}} := M_{\text{Maxover}} \cdot \text{kNm}$   $F_{\text{fat.max}} := F_{\text{Fatmax}} \cdot \text{kN}$

$M_{\text{fat.min}} := M_{\text{Minover}} \cdot \text{kNm}$   $F_{\text{fat.min}} := F_{\text{Fatmin}} \cdot \text{kN}$

$i := 0 \dots (\text{length}(M_{\text{fat.max}}) - 1)$

**A first check of the fatigue stresses shows that the horizontal force is small in comparison to the overturning moment and has no impact on the fatigue life of the slab. The horizontal force is therefore disregarded in the fatigue assessment.**

$$b_{2w} := \frac{D}{2} - r_{\text{average}} = 6.06 \text{ m}$$

$$b_{2w} := \frac{D}{2} + r_{\text{average}} = 9.94 \text{ m}$$

To find the stresses that are affecting the fatigue life of the slab 4 different strut and tie models are established. Four models are needed in order to describe the behaviour of the slab properly since the fatigue moments are of varying size and results in differences in the location of the reaction force from the ground. To simplify the calculation procedure the force couple  $F_C$  and  $F_T$  are assumed to be acting on the anchor ring with a distance between them of  $2r_{\text{average}}$ .

## C.2 Maximum stresses in struts and ties

$$F_{C.\max_1} := \frac{F_z}{2} + \left( \frac{M_{\text{fat}.\max_1}}{2 \cdot r_{\text{average}}} \right) = \dots$$

$$F_{T.\max_1} := \frac{F_z}{2} - \frac{M_{\text{fat}.\max_1}}{2 \cdot r_{\text{average}}} = \dots$$

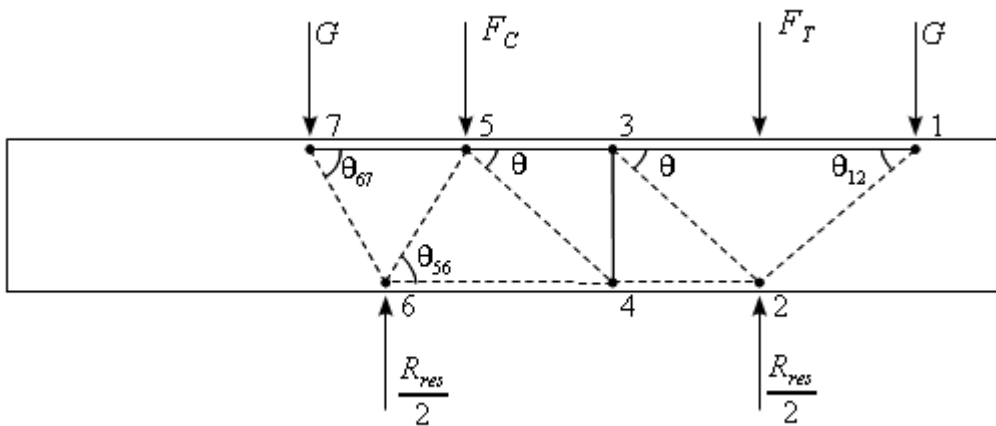
$$R_{\text{res}.\max_1} := F_{T.\max_1} + F_{C.\max_1} + G_{\text{SLS}} = \dots$$

$$G_{\text{SLS}} := \frac{G_{\text{SLS}}}{2} = 6.4 \cdot \text{MN}$$

$$b_i := \frac{(F_z + G_{\text{SLS}}) \cdot \frac{D}{2} - M_{\text{fat}.\max_1}}{R_{\text{res}.\max_1}} = \dots$$

$$u_{\text{node}} = 0.174 \text{ m}$$

**Strut and tie model 1, when the resultant of the reaction force is acting inside the anchor ring.**



$$\theta_{12_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{3 \frac{D}{4} - b_3} \right] = \dots \cdot \text{deg}$$



$$\theta_i := \text{atan}\left[\frac{(d_2 - d'_2)}{r_{\text{average}}}\right] = \dots \cdot \text{deg}$$

$$\theta_{56_i} := \text{atan}\left[\frac{(d_2 - d'_2)}{\left(\frac{D}{2} - r_{\text{average}}\right) - (2 \cdot b_i - b_3)}\right] = \dots \cdot \text{deg}$$

$$\theta_{67_i} := \text{atan}\left[\frac{(d_2 - d'_2)}{(2b_i - b_3) - \frac{D}{4}}\right] = \dots \cdot \text{deg}$$

#### Node 1

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})} = \dots \cdot \text{MN}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i} = \dots \cdot \text{MN}$$

#### Node 2

$$C_{2.3_i} := \frac{\frac{R_{\text{res.max}_1}}{2} - F_{T.\text{max}_1} - C_{1.2_i} \cdot \sin(\theta_{12_i})}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{2.4_i} := -\cos(\theta_i) \cdot C_{2.3_i} + \cos(\theta_{12_i}) C_{1.2_i} = \dots \cdot \text{MN}$$

#### Node 3

$$T_{3.5_i} := -\cos(\theta_i) \cdot C_{2.3_i} + T_{1.3_i} = \dots \cdot \text{MN}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i} = \dots \cdot \text{MN}$$

#### Node 4

$$C_{4.5_i} := \frac{T_{3.4_i}}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{4.6_i} := C_{2.4_i} - \cos(\theta_i) \cdot C_{4.5_i} = \dots \cdot \text{MN}$$

#### Node 5

$$C_{5.6_i} := \frac{F_{C.\text{max}_1} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})} = \dots \cdot \text{MN}$$

$$T_{5.7_i} := T_{3.5_i} - \cos(\theta_i) \cdot C_{4.5_i} + \cos(\theta_{56_i}) \cdot C_{5.6_i} = \dots \cdot \text{MN}$$

**Node 6**

$$C_{6.7_i} := \frac{\frac{R_{res.max_1}}{2} - C_{5.6_i} \cdot \sin(\theta_{56_i})}{\sin(\theta_{67_i})} = \dots$$

$$C_{4.6_i} = \cos(\theta_{67_i}) \cdot C_{6.7_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

**Node 7**

$$C_{6.7_i} = \frac{T_{5.7_i}}{\cos(\theta_{67_i})}$$

$$\sin(\theta_{67_i}) \cdot C_{6.7_i} = G$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := (-T_{3.5_i}) + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5_i} := 2 \cdot u_{node} \cdot k_{average} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5_i}}$$

Maximum stresses from strut and tie model 1

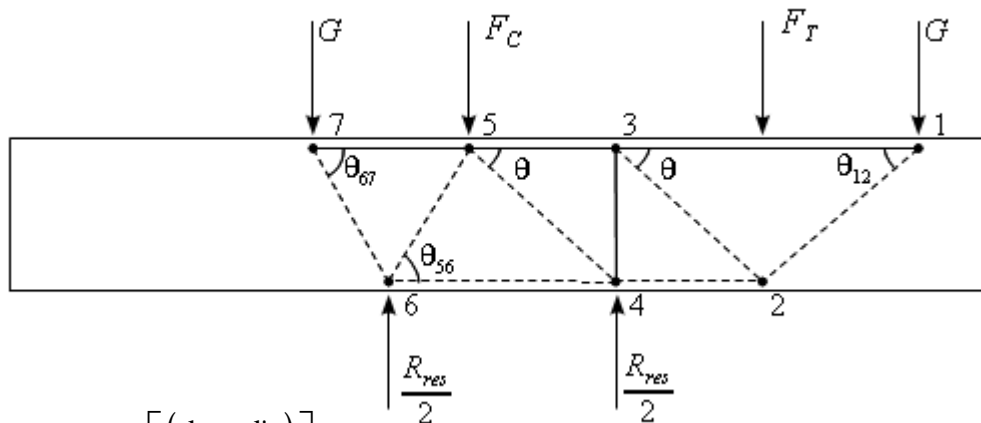
$$\sigma_{cc.1.max_i} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.1.max_i} := \begin{cases} 0 & \text{if } C_{4.6_i} \geq 0 \\ \frac{(-C_{4.6_i})}{A_s} & \text{if } C_{4.6_i} < 0 \end{cases}$$

$$\sigma_{st'.1.max_i} := \begin{cases} \frac{T_{3.5_i}}{A'_s} & \text{if } T_{3.5_i} \geq 0 \\ 0 & \text{if } T_{3.5_i} < 0 \end{cases}$$

$$V_{1.max_i} := T_{3.4_i}$$

**Strut and tie model 2, when the resultant of the reaction force is acting inside the anchor ring.**



$$\theta_{12_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{3 \frac{D}{4} - b_3} \right] = \dots \cdot \text{deg}$$

$$\theta_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{r_{\text{average}}} \right] = \dots \cdot \text{deg}$$

$$\theta_{56_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left( \frac{D}{2} - r_{\text{average}} \right) - \left( 2 \cdot b_i - \frac{D}{2} \right)} \right] = \dots \cdot \text{deg}$$

$$\theta_{67_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left( 2b_i - \frac{D}{2} \right) - \frac{D}{4}} \right] = \dots \cdot \text{deg}$$

### Node 1

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})} = \dots \cdot \text{MN}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i} = \dots \cdot \text{MN}$$

### Node 2

$$C_{2.3_i} := \frac{(-F_{T.\text{max}})_i - C_{1.2_i} \cdot \sin(\theta_{12_i})}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{2.4_i} := -\cos(\theta_i) \cdot C_{2.3_i} + \cos(\theta_{12_i}) C_{1.2_i} = \dots \cdot \text{MN}$$

### Node 3

$$T_{3.5_i} := -\cos(\theta_i) \cdot C_{2.3_i} + T_{1.3_i} = \dots \cdot \text{MN}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i} = \dots \cdot \text{MN}$$

**Node 4**

$$C_{4.5_i} := \frac{T_{3.4_i} + \frac{R_{\text{res.max}_1}}{2}}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{4.6_i} := C_{2.4_i} - \cos(\theta_i) \cdot C_{4.5_i} = \dots \cdot \text{MN}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.\text{max}_1} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})} = \dots \cdot \text{MN}$$

$$T_{5.7_i} := T_{3.5_i} - \cos(\theta_i) \cdot C_{4.5_i} + \cos(\theta_{56_i}) \cdot C_{5.6_i} = \dots \cdot \text{MN}$$

**Node 6**

$$C_{6.7_i} := \frac{\frac{R_{\text{res.max}_1}}{2} - C_{5.6_i} \cdot \sin(\theta_{56_i})}{\sin(\theta_{67_i})} = \dots$$

$$C_{4.6_i} = \cos(\theta_{67_i}) \cdot C_{6.7_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

**Node 7**

$$C_{6.7_i} = \frac{T_{5.7_i}}{\cos(\theta_{67_i})}$$

$$\sin(\theta_{67_i}) \cdot C_{6.7_i} = G$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := (-T_{3.5_i}) + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5} := 2 \cdot u_{\text{node}} \cdot k_{\text{average}} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5}}$$

Maximum stresses from strut and tie model 2

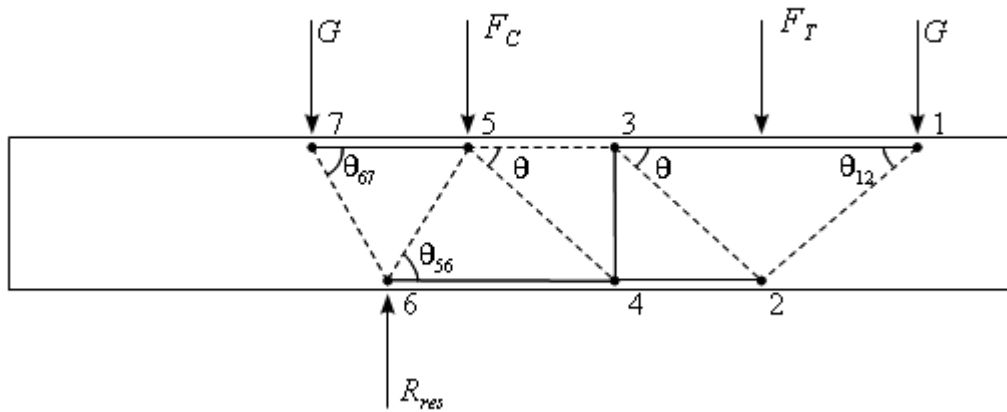
$$\sigma_{cc.2.\text{max}_1} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.2.max_1} := \begin{cases} 0 & \text{if } C_{4.6_i} \geq 0 \\ \frac{(-C_{4.6})_i}{A_s} & \text{if } C_{4.6_i} < 0 \end{cases}$$

$$\sigma_{st'.2.max_1} := \begin{cases} \frac{T_{3.5_i}}{A'_s} & \text{if } T_{3.5_i} \geq 0 \\ 0 & \text{if } T_{3.5_i} < 0 \end{cases}$$

$$V_{2.max_1} := \begin{cases} T_{3.4_i} & \text{if } T_{3.4_i} \geq 0 \\ 0 & \text{if } T_{3.4_i} < 0 \end{cases}$$

**Strut and tie model 3, when the reaction force from the ground is acting outside the anchor ring and not further away from it than node 7.**



$$\theta_{12_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(3 \frac{D}{4} - b_3\right)} \right]$$

$$\theta_{56_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left[\left(\frac{D}{2} - r_{\text{average}}\right) - b_i\right]} \right]$$

$$\theta_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{(r_{\text{average}})} \right]$$

$$\theta_{67_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(b_i - \frac{D}{4}\right)} \right]$$

**Node 1**

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 2**

$$C_{2.3_i} := \frac{(-F_{T.\max})_i - \sin(\theta_{12_i}) C_{1.2_i}}{\sin(\theta_i)}$$

$$T_{2.4_i} := \cos(\theta_i) \cdot C_{2.3_i} - \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 3**

$$C_{3.5_i} := \cos(\theta_i) \cdot C_{2.3_i} - T_{1.3_i}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i}$$

**Node 4**

$$C_{4.5_i} := \frac{T_{3.4_i}}{\sin(\theta_i)}$$

$$T_{4.6_i} := T_{2.4_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.\max_i} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})}$$

$$T_{5.7_i} := \cos(\theta_{56_i}) \cdot C_{5.6_i} - \cos(\theta_i) \cdot C_{4.5_i} - C_{3.5_i}$$

**Node 6**

$$C_{6.7_i} := \frac{R_{\text{res.max}_i} - \sin(\theta_{56_i}) \cdot C_{5.6_i}}{\sin(\theta_{67_i})}$$

$$T_{4.6_i} = \cos(\theta_{56_i}) \cdot C_{5.6_i} - \cos(\theta_{67_i}) \cdot C_{6.7_i}$$

**Node 7**

$$G = \sin(\theta_{67_i}) \cdot C_{6.7_i}$$

$$T_{5.7_i} = \cos(\theta_{67_i}) \cdot C_{6.7_i}$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := C_{3.5_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5_i} := 2 \cdot u_{\text{node}} \cdot k_{\text{average}} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5}}$$

Maximum stresses from strut and tie model 3

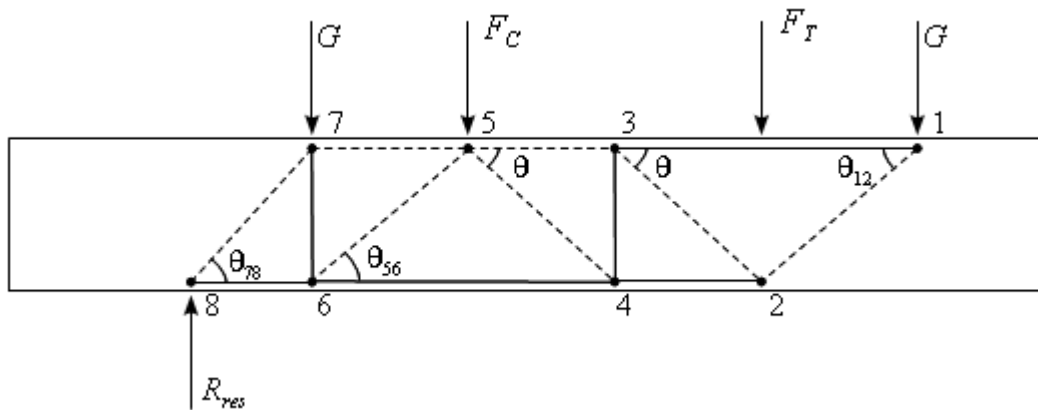
$$\sigma_{cc.3.\max_i} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.3.\max_i} := \begin{cases} 0 & \text{if } T_{4.6_i} < 0 \\ \frac{T_{4.6_i}}{A_s} & \text{if } T_{4.6_i} \geq 0 \end{cases}$$

$$\sigma_{st'.3.\max_i} := \begin{cases} \frac{(-C_{3.5_i})}{A'_s} & \text{if } C_{3.5_i} \leq 0 \\ 0 & \text{if } C_{3.5_i} > 0 \end{cases}$$

$$V_{3.\max_i} := \begin{cases} T_{3.4_i} & \text{if } T_{3.4_i} \geq 0 \\ 0 & \text{if } T_{3.4_i} < 0 \end{cases}$$

**Strut and tie model 4, when the reaction force from the ground is acting outside the anchor ring and further away from it than node 7.**



$$\theta_{12_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(3 \frac{D}{4} - b_3\right)} \right]$$

$$\theta_{56_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(\frac{D}{4} - r_{\text{average}}\right)} \right]$$

$$\theta_{78_i} := \text{atan} \left( \frac{(d_2 - d'_2)}{\frac{D}{4} - b_i} \right)$$

$$\theta_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{(r_{\text{average}})} \right]$$

**Node 1**

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 2**

$$C_{2.3_i} := \frac{(-F_{T.\max})_i - \sin(\theta_{12_i}) C_{1.2_i}}{\sin(\theta_i)}$$

$$T_{2.4_i} := \cos(\theta_i) \cdot C_{2.3_i} - \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 3**

$$C_{3.5_i} := \cos(\theta_i) \cdot C_{2.3_i} - T_{1.3_i}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i}$$

**Node 4**

$$C_{4.5_i} := \frac{T_{3.4_i}}{\sin(\theta_i)}$$

$$T_{4.6_i} := T_{2.4_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.\max_i} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})}$$

$$C_{5.7_i} := C_{3.5_i} + \cos(\theta_i) \cdot C_{4.5_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$



**Node 6**

$$T_{6.8_i} := T_{4.6_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

$$T_{6.7_i} := \sin(\theta_{56_i}) \cdot C_{5.6_i}$$

**Node 7**

$$C_{7.8_i} := \frac{C_{5.7_i}}{\cos(\theta_{78_i})}$$

$$T_{6.7_i} = \sin(\theta_{78_i}) \cdot C_{7.8_i} - G$$

**Node 8**

$$T_{6.8_i} = \cos(\theta_{78_i}) \cdot C_{7.8_i}$$

$$R_{\text{res.max}_i} = \sin(\theta_{78_i}) \cdot C_{7.8_i}$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := C_{3.5_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5_i} := 2 \cdot u_{\text{node}} \cdot k_{\text{average}} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5_i}}$$

Maximum stresses from strut and tie model 4

$$\sigma_{cc.4.\text{max}_i} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.4.\text{max}_i} := \begin{cases} 0 & \text{if } T_{4.6_i} < 0 \\ \frac{T_{4.6_i}}{A_s} & \text{if } T_{4.6_i} \geq 0 \end{cases}$$

$$\sigma_{st'.4.\text{max}_i} := \begin{cases} \frac{(-C_{3.5_i})}{A'_s} & \text{if } C_{3.5_i} \leq 0 \\ 0 & \text{if } C_{3.5_i} > 0 \end{cases}$$

$$V_{4.\text{max}_i} := \begin{cases} T_{3.4_i} & \text{if } T_{3.4_i} \geq 0 \\ 0 & \text{if } T_{3.4_i} < 0 \end{cases}$$

### Maximum stresses in critical sections

$$\sigma_{cc.max_1} := \begin{cases} \sigma_{cc.1.max_1} & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{cc.2.max_1} & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{cc.3.max_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ \sigma_{cc.4.max_1} & \text{otherwise} \end{cases}$$

$$\sigma_{st.max_1} := \begin{cases} \sigma_{st.1.max_1} \cdot 0 & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st.2.max_1} \cdot 0 & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st.3.max_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ \sigma_{st.4.max_1} & \text{otherwise} \end{cases}$$

$$\sigma_{st'.max_1} := \begin{cases} \sigma_{st'.1.max_1} & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st'.2.max_1} & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st'.3.max_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ \sigma_{st'.4.max_1} & \text{otherwise} \end{cases}$$

$$V_{max_1} := \begin{cases} V_{1.max_1} & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ V_{2.max_1} & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ V_{3.max_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ V_{4.max_1} & \text{otherwise} \end{cases}$$

### C.3 Minimum stresses in struts and ties

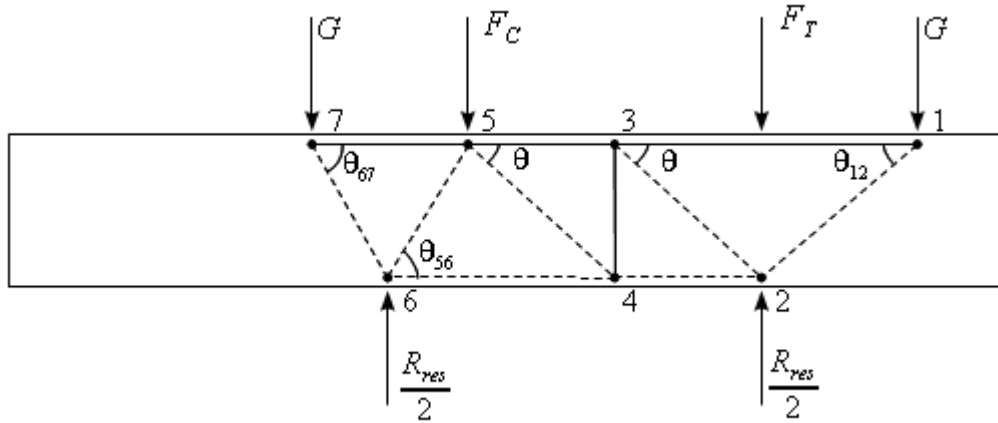
$$F_{C.min_1} := \frac{F_Z}{2} + \left( \frac{M_{fat.min_1}}{2 \cdot r_{average}} \right) = \dots$$

$$F_{T.min_1} := \frac{F_Z}{2} - \frac{M_{fat.min_1}}{2 \cdot r_{average}} = \dots$$

$$R_{res.min_1} := F_{T.min_1} + F_{C.min_1} + G_{SLS} = \dots$$

$$b_i := \frac{\left( F_Z + G_{SLS} \right) \cdot \frac{D}{2} - M_{fat.min_1}}{R_{res.min_1}} = \dots$$

**Strut and tie model 1, when the resultant of the reaction force is acting inside the anchor ring.**



$$\theta_{12_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{3 \frac{D}{4} - b_3} \right] = \dots \cdot \text{deg}$$

$$\theta_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{r_{average}} \right] = \dots \cdot \text{deg}$$

$$\theta_{56_i} := \begin{cases} (90 \cdot \text{deg}) & \text{if } \left( \frac{D}{2} - r_{average} \right) - (2 \cdot b_i - b_3) = 0 \\ \text{atan} \left[ \frac{(d_2 - d'_2)}{\left( \frac{D}{2} - r_{average} \right) - (2 \cdot b_i - b_3)} \right] & \text{otherwise} \end{cases} = \dots \cdot \text{deg}$$

$$\theta_{67_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left( (2b_i - b_3) - \frac{D}{4} \right)} \right] = \dots \cdot \text{deg}$$

**Node 1**

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})} = \dots \cdot \text{MN}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i} = \dots \cdot \text{MN}$$

**Node 2**

$$C_{2.3_i} := \frac{\frac{R_{\text{res.min}_1}}{2} - F_{T.\text{min}_1} - C_{1.2_i} \cdot \sin(\theta_{12_i})}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{2.4_i} := -\cos(\theta_i) \cdot C_{2.3_i} + \cos(\theta_{12_i}) C_{1.2_i} = \dots \cdot \text{MN}$$

**Node 3**

$$T_{3.5_i} := -\cos(\theta_i) \cdot C_{2.3_i} + T_{1.3_i} = \dots \cdot \text{MN}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i} = \dots \cdot \text{MN}$$

**Node 4**

$$C_{4.5_i} := \frac{T_{3.4_i}}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{4.6_i} := C_{2.4_i} - \cos(\theta_i) \cdot C_{4.5_i} = \dots \cdot \text{MN}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.\text{min}_1} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})} = \dots \cdot \text{MN}$$

$$T_{5.7_i} := T_{3.5_i} - \cos(\theta_i) \cdot C_{4.5_i} + \cos(\theta_{56_i}) \cdot C_{5.6_i} = \dots \cdot \text{MN}$$

**Node 6**

$$C_{6.7_i} := \frac{\frac{R_{\text{res.min}_1}}{2} - C_{5.6_i} \cdot \sin(\theta_{56_i})}{\sin(\theta_{67_i})} = \dots$$

$$C_{4.6_i} = \cos(\theta_{67_i}) \cdot C_{6.7_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

### Node 7

$$C_{6.7_i} = \frac{T_{5.7_i}}{\cos(\theta_{67_i})}$$

$$\sin(\theta_{67_i}) \cdot C_{6.7_i} = G$$

### Compressive stress under the compressive force $F_C$ (node 5)

$$C_{c0.5_i} := (-T_{3.5})_i + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5_i} := 2 \cdot u_{\text{node}} \cdot k_{\text{average}} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5_i}}$$

Maximum stresses from strut and tie model 1

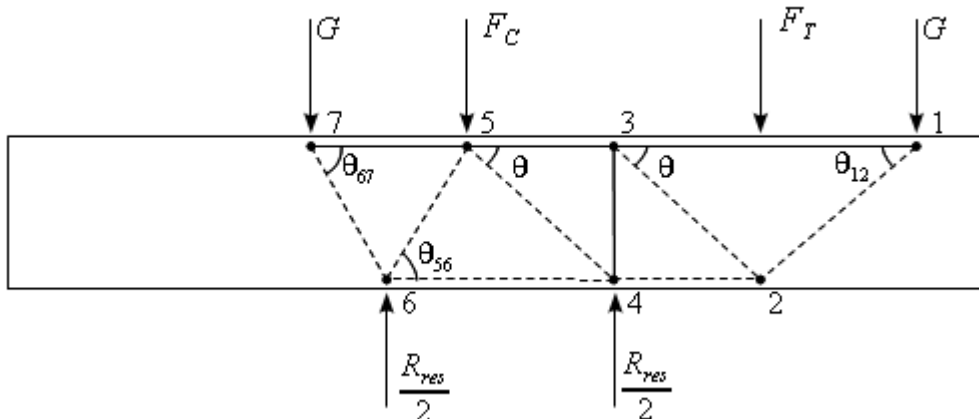
$$\sigma_{cc.1.min_i} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.1.min_i} := \begin{cases} 0 & \text{if } C_{4.6_i} \geq 0 \\ \frac{(-C_{4.6})_i}{A_s} & \text{if } C_{4.6_i} < 0 \end{cases}$$

$$\sigma_{st'.1.min_i} := \begin{cases} \frac{T_{3.5_i}}{A'_s} & \text{if } T_{3.5_i} \geq 0 \\ 0 & \text{if } T_{3.5_i} < 0 \end{cases}$$

$$V_{1.min_i} := T_{3.4_i}$$

**Strut and tie model 2, when the resultant of the reaction force is acting inside the anchor ring.**



$$\theta_{12_i} := \operatorname{atan}\left[\frac{(d_2 - d'_2)}{3\frac{D}{4} - b_3}\right] = \dots \cdot \deg$$

$$\theta_i := \operatorname{atan}\left[\frac{(d_2 - d'_2)}{r_{\text{average}}}\right] = \dots \cdot \deg$$

$$\theta_{56_i} := \operatorname{atan}\left[\frac{(d_2 - d'_2)}{\left(\frac{D}{2} - r_{\text{average}}\right) - \left(2 \cdot b_i - \frac{D}{2}\right)}\right] = \dots \cdot \deg$$

$$\theta_{67_i} := \operatorname{atan}\left[\frac{(d_2 - d'_2)}{\left(2b_i - \frac{D}{2}\right) - \frac{D}{4}}\right] = \dots \cdot \deg$$

#### Node 1

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})} = \dots \cdot \text{MN}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i} = \dots \cdot \text{MN}$$

#### Node 2

$$C_{2.3_i} := \frac{(-F_{T.\min})_i - C_{1.2_i} \cdot \sin(\theta_{12_i})}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{2.4_i} := -\cos(\theta_i) \cdot C_{2.3_i} + \cos(\theta_{12_i}) C_{1.2_i} = \dots \cdot \text{MN}$$

#### Node 3

$$T_{3.5_i} := -\cos(\theta_i) \cdot C_{2.3_i} + T_{1.3_i} = \dots \cdot \text{MN}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i} = \dots \cdot \text{MN}$$

#### Node 4

$$C_{4.5_i} := \frac{T_{3.4_i} + \frac{R_{\text{res.min}}}{2}}{\sin(\theta_i)} = \dots \cdot \text{MN}$$

$$C_{4.6_i} := C_{2.4_i} - \cos(\theta_i) \cdot C_{4.5_i} = \dots \cdot \text{MN}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.min_1} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})} = \dots \cdot MN$$

$$T_{5.7_i} := T_{3.5_i} - \cos(\theta_i) \cdot C_{4.5_i} + \cos(\theta_{56_i}) \cdot C_{5.6_i} = \dots \cdot MN$$

**Node 6**

$$C_{6.7_i} := \frac{\frac{R_{res.min_1}}{2} - C_{5.6_i} \cdot \sin(\theta_{56_i})}{\sin(\theta_{67_i})} = \dots$$

$$C_{4.6_i} = \cos(\theta_{67_i}) \cdot C_{6.7_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

**Node 7**

$$C_{6.7_i} = \frac{T_{5.7_i}}{\cos(\theta_{67_i})}$$

$$\sin(\theta_{67_i}) \cdot C_{6.7_i} = G$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := (-T_{3.5_i})_i + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5} := 2 \cdot u_{node} \cdot k_{average} = 1.312 m^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5}}$$

Maximum stresses from strut and tie model 2

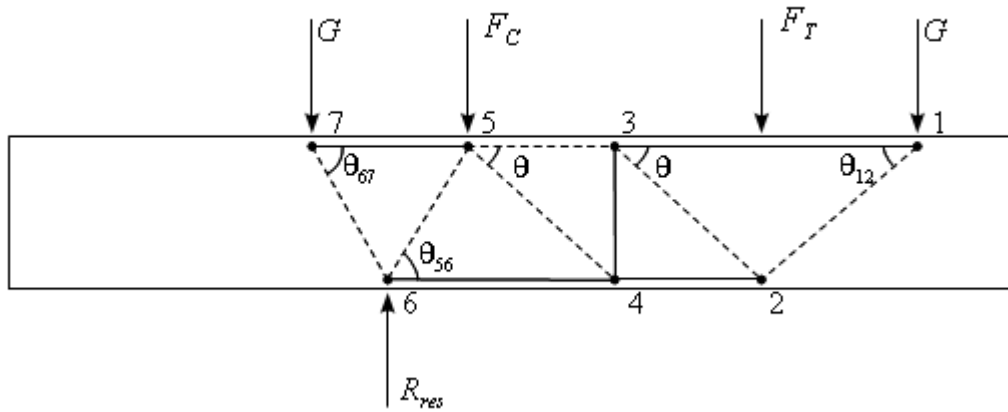
$$\sigma_{cc.2.min_1} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.2.min_1} := \begin{cases} 0 & \text{if } C_{4.6_i} \geq 0 \\ \frac{(-C_{4.6_i})_i}{A_s} & \text{if } C_{4.6_i} < 0 \end{cases}$$

$$\sigma_{st'.2.min_1} := \begin{cases} \frac{T_{3.5_i}}{A'_s} & \text{if } T_{3.5_i} \geq 0 \\ 0 & \text{if } T_{3.5_i} < 0 \end{cases}$$

$$V_{2.min_i} := \begin{cases} T_{3.4_i} & \text{if } T_{3.4_i} \geq 0 \\ 0 & \text{if } T_{3.4_i} < 0 \end{cases}$$

**Strut and tie model 3, when the reaction force from the ground is outside the anchor ring and not further away from it than node 7.**



$$\theta_{12_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(3 \frac{D}{4} - b_3\right)} \right]$$

$$\theta_{56_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left[\left(\frac{D}{2} - r_{\text{average}}\right) - b_i\right]} \right]$$

$$\theta_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{(r_{\text{average}})} \right]$$

$$\theta_{67_i} := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(b_i - \frac{D}{4}\right)} \right]$$

**Node 1**

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 2**

$$C_{2.3_i} := \frac{(-F_{T.min_i}) - \sin(\theta_{12_i}) C_{1.2_i}}{\sin(\theta_i)}$$

$$T_{2.4_i} := \cos(\theta_i) \cdot C_{2.3_i} - \cos(\theta_{12_i}) \cdot C_{1.2_i}$$



**Node 3**

$$C_{3.5_i} := \cos(\theta_i) \cdot C_{2.3_i} - T_{1.3_i}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i}$$

**Node 4**

$$C_{4.5_i} := \frac{T_{3.4_i}}{\sin(\theta_i)}$$

$$T_{4.6_i} := T_{2.4_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.min_i} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})}$$

$$T_{5.7_i} := \cos(\theta_{56_i}) \cdot C_{5.6_i} - \cos(\theta_i) \cdot C_{4.5_i} - C_{3.5_i}$$

**Node 6**

$$C_{6.7_i} := \frac{R_{res.min_i} - \sin(\theta_{56_i}) \cdot C_{5.6_i}}{\sin(\theta_{67_i})}$$

$$T_{4.6_i} = \cos(\theta_{56_i}) \cdot C_{5.6_i} - \cos(\theta_{67_i}) \cdot C_{6.7_i}$$

**Node 7**

$$G = \sin(\theta_{67_i}) \cdot C_{6.7_i}$$

$$T_{5.7_i} = \cos(\theta_{67_i}) \cdot C_{6.7_i}$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := C_{3.5_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5} := 2 \cdot u_{node} \cdot k_{average} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5}}$$

Maximum stresses from strut and tie model 3

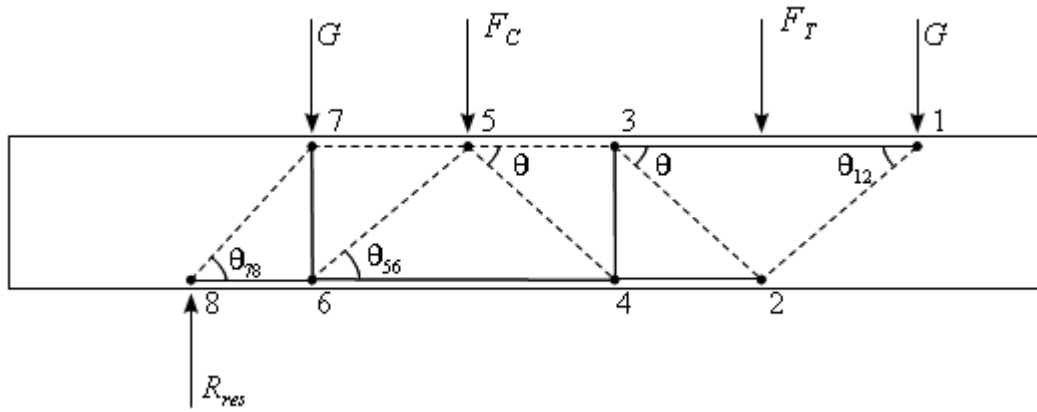
$$\sigma_{cc.3.min_i} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.3.min}_i := \begin{cases} 0 & \text{if } T_{4.6}_i < 0 \\ \frac{T_{4.6}_i}{A_s} & \text{if } T_{4.6}_i \geq 0 \end{cases}$$

$$\sigma_{st'.3.min}_i := \begin{cases} \frac{(-C_{3.5})_i}{A'_s} & \text{if } C_{3.5}_i \leq 0 \\ 0 & \text{if } C_{3.5}_i > 0 \end{cases}$$

$$V_{3.min}_i := \begin{cases} T_{3.4}_i & \text{if } T_{3.4}_i \geq 0 \\ 0 & \text{if } T_{3.4}_i < 0 \end{cases}$$

**Strut and tie model 4, when the reaction force from the ground is acting outside the anchor ring and further away from it than node 7.**



$$\theta_{12}_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(3 \frac{D}{4} - b_3\right)} \right]$$

$$\theta_{56}_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{\left(\frac{D}{4} - r_{\text{average}}\right)} \right]$$

$$\theta_{78}_i := \text{atan} \left( \frac{d_2 - d'_2}{\frac{D}{4} - b_i} \right)$$

$$\theta_i := \text{atan} \left[ \frac{(d_2 - d'_2)}{(r_{\text{average}})} \right]$$

**Node 1**

$$C_{1.2_i} := \frac{G}{\sin(\theta_{12_i})}$$

$$T_{1.3_i} := \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 2**

$$C_{2.3_i} := \frac{(-F_{T.min_i} - \sin(\theta_{12_i}) C_{1.2_i})}{\sin(\theta_i)}$$

$$T_{2.4_i} := \cos(\theta_i) \cdot C_{2.3_i} - \cos(\theta_{12_i}) \cdot C_{1.2_i}$$

**Node 3**

$$C_{3.5_i} := \cos(\theta_i) \cdot C_{2.3_i} - T_{1.3_i}$$

$$T_{3.4_i} := \sin(\theta_i) \cdot C_{2.3_i}$$

**Node 4**

$$C_{4.5_i} := \frac{T_{3.4_i}}{\sin(\theta_i)}$$

$$T_{4.6_i} := T_{2.4_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

**Node 5**

$$C_{5.6_i} := \frac{F_{C.min_i} - \sin(\theta_i) \cdot C_{4.5_i}}{\sin(\theta_{56_i})}$$

$$C_{5.7_i} := C_{3.5_i} + \cos(\theta_i) \cdot C_{4.5_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

**Node 6**

$$T_{6.8_i} := T_{4.6_i} - \cos(\theta_{56_i}) \cdot C_{5.6_i}$$

$$T_{6.7_i} := \sin(\theta_{56_i}) \cdot C_{5.6_i}$$

**Node 7**

$$C_{7.8_i} := \frac{C_{5.7_i}}{\cos(\theta_{78_i})}$$

$$T_{6.7_i} = \sin(\theta_{78_i}) \cdot C_{7.8_i} - G$$

**Node 8**

$$T_{6.8_i} = \cos(\theta_{78_i}) \cdot C_{7.8_i}$$

$$R_{\text{res.min}_i} = \sin(\theta_{78_i}) \cdot C_{7.8_i}$$

**Compressive stress under the compressive force  $F_C$  (node 5)**

$$C_{c0.5_i} := C_{3.5_i} + \cos(\theta_i) \cdot C_{4.5_i}$$

$$A_{c0.5_i} := 2 \cdot u_{\text{node}} \cdot k_{\text{average}} = 1.312 \text{ m}^2$$

$$\sigma_{c0.5_i} := \frac{C_{c0.5_i}}{A_{c0.5_i}}$$

Maximum stresses from strut and tie model 4

$$\sigma_{cc.4.min_i} := \begin{cases} 0 & \text{if } \sigma_{c0.5_i} < 0 \\ \sigma_{c0.5_i} & \text{if } \sigma_{c0.5_i} \geq 0 \end{cases}$$

$$\sigma_{st.4.min_i} := \begin{cases} 0 & \text{if } T_{4.6_i} < 0 \\ \frac{T_{4.6_i}}{A_s} & \text{if } T_{4.6_i} \geq 0 \end{cases}$$

$$\sigma_{st'.4.min_i} := \begin{cases} \frac{(-C_{3.5_i})}{A'_s} & \text{if } C_{3.5_i} \leq 0 \\ 0 & \text{if } C_{3.5_i} > 0 \end{cases}$$

$$V_{4.min_i} := \begin{cases} T_{3.4_i} & \text{if } T_{3.4_i} \geq 0 \\ 0 & \text{if } T_{3.4_i} < 0 \end{cases}$$

**Maximum stresses in critical sections**

$$\sigma_{cc.min_i} := \begin{cases} \sigma_{cc.1.min_i} & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{cc.2.min_i} & \text{if } \frac{D}{2} - r_{\text{average}} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{cc.3.min_i} & \text{if } \frac{D}{2} - r_{\text{average}} \geq b_i > \frac{D}{4} \\ \sigma_{cc.4.min_i} & \text{otherwise} \end{cases}$$

$$\sigma_{cc.min.EC2_1} := \begin{cases} \sigma_{cc.min_1} & \text{if } \sigma_{cc.max_1} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\sigma_{st.min_1} := \begin{cases} \sigma_{st.1.min_1} \cdot 0 & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st.2.min_1} \cdot 0 & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st.3.min_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ \sigma_{st.4.min_1} & \text{otherwise} \end{cases}$$

$$\sigma_{st'.min_1} := \begin{cases} \sigma_{st'.1.min_1} & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st'.2.min_1} & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ \sigma_{st'.3.min_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ \sigma_{st'.4.min_1} & \text{otherwise} \end{cases}$$

$$V_{min_1} := \begin{cases} V_{1.min_1} & \text{if } b_i \geq \frac{b_3}{2} + \frac{D}{8} \\ V_{2.min_1} & \text{if } \frac{D}{2} - r_{average} \leq b_i < \frac{b_3}{2} + \frac{D}{8} \\ V_{3.min_1} & \text{if } \frac{D}{2} - r_{average} \geq b_i > \frac{D}{4} \\ V_{4.min_1} & \text{otherwise} \end{cases}$$

## D. Fatigue Assessment

### D.1 Fatigue assessment of reinforcing steel

#### Steel stress range in bottom reinforcement

$$\Delta\sigma_{st_i} := \sigma_{st,max_i} - \sigma_{st,min_i}$$

$$\Delta\sigma_{st_i} := \left| \Delta\sigma_{st_i} \right|$$

#### Steel stress range in top reinforcement

$$\Delta\sigma_{st'_i} := \sigma_{st',min_i} - \sigma_{st',max_i}$$

$$\Delta\sigma_{st'_i} := \begin{cases} \left| \Delta\sigma_{st'_i} \right| & \text{if } \Delta\sigma_{st'_i} > 0 \\ \left| \sigma_{st',1,min_i} - \sigma_{st',1,max_i} \right| & \text{otherwise} \end{cases}$$

#### Fatigue assessment of reinforcing steel, according to Eurocode 2 [EN 1992-1-1:2005 6.8.4]

$\gamma_{Sfat} := 1.15$  Partial factor that takes uncertainties in the material into account,  
[EN 1992-1-1:2005 2.4.2.4 table 2.1N]

$\gamma_{Ffat} := 1.0$  Partial factor for fatigue loading,  
[EN 1992-1-1:2005 2.4.2.3 (1)]

$\Delta\sigma_{Rsk} := 162.5\text{MPa}$  The fatigue stress range after  $N^*$  cycles  
[EN 1992-1-1:2005 table 6.3N]

$k_1$  is the exponent that defines the slope of the first part of the S-N curve  
[EN 1992-1-1:2005 table 6.3N].

$$k_1 := 5$$

$k_2$  is the exponent that defines the slope of the second part of the S-N curve.

$$k_2 := 9$$

#### Fatigue assessment of the bottom reinforcement

Reference value for number of cycles until fatigue failure, depending on reinforcement type  
[EN 1992-1-1:2005 table 6.3N]

$$N_{ref} := 10^6$$

Number of cycles until fatigue failure, overturning moment.

$$i := 0 \dots (\text{length}(\Delta\sigma_{st}) - 1)$$

$$N_{st_i} := \begin{cases} N_{ref} \cdot \left[ \frac{\left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right)}{\gamma Ffat \cdot \Delta\sigma_{st_i}} \right]^{k_1} & \text{if } \gamma Ffat \cdot \Delta\sigma_{st_i} \geq \left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right) \wedge \Delta\sigma_{st_i} \neq 0 \\ N_{ref} \cdot \left[ \frac{\left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right)}{\gamma Ffat \cdot \Delta\sigma_{st_i}} \right]^{k_2} & \text{if } \gamma Ffat \cdot \Delta\sigma_{st_i} < \left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right) \wedge \Delta\sigma_{st_i} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Total fatigue damage of the bottom reinforcement, caused by overturning moment, according to the Palmgren-Miner rule.

$$j := (\text{length}(N_{st}) - 1)$$

$$D_{st} := \sum_{i=0}^j \left( \begin{cases} 0 & \text{if } N_{st_i} = 0 \\ \frac{n_{over_i}}{N_{st_i}} & \text{otherwise} \end{cases} \right)$$

$$D_{st} \leq 1 = 1$$

$$D_{st} = 0.00254073$$

### Fatigue assessment of the top reinforcement

Reference value for number of cycles until fatigue failure, depending on reinforcement type [EN 1992-1-1:2005 table 6.3N]

$$N_{ref} := 10^6$$

Number of cycles until fatigue failure, overturning moment.

$$i := 0..(\text{length}(\Delta\sigma_{st'}) - 1)$$

$$N_{st'_i} := \begin{cases} N_{ref} \cdot \left[ \frac{\left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right)}{\gamma Ffat \cdot \Delta\sigma_{st'_i}} \right]^{k_1} & \text{if } \gamma Ffat \cdot \Delta\sigma_{st'_i} \geq \left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right) \wedge \Delta\sigma_{st'_i} \neq 0 \\ N_{ref} \cdot \left[ \frac{\left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right)}{\gamma Ffat \cdot \Delta\sigma_{st'_i}} \right]^{k_2} & \text{if } \gamma Ffat \cdot \Delta\sigma_{st'_i} < \left( \frac{\Delta\sigma}{\gamma Sfat} Rsk \right) \wedge \Delta\sigma_{st'_i} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Total fatigue damage of the top reinforcement, caused by overturning moment, according to the Palmgren-Miner rule.

$$j_v := (\text{length}(N_{st'}) - 1)$$

$$D_{st'} := \sum_{i=0}^j \left( \begin{cases} 0 & \text{if } N_{st'_i} = 0 \\ \frac{n_{over_i}}{N_{st'_i}} & \text{otherwise} \end{cases} \right)$$

$$D_{st'} \leq 1 = 0$$

$$D_{st'} = 62.155$$

### **Fatigue assessment of reinforcing steel, according to Eurocode 2 [EN 1992-1-1:2005 6.8.5]**

The following condition should be fulfilled for required fatigue resistance

$$\gamma_{Ffat} \cdot \Delta\sigma_{s,max} \leq \frac{\Delta\sigma_{Rsk}}{\gamma_{Sfat}}$$

Bottom reinforcement

Maximum steel stress range in the bottom reinforcement

$$\Delta\sigma_{st,max} := \max(\Delta\sigma_{st}) = 234.207 \cdot \text{MPa}$$

Check of condition

$$\gamma_{Ffat} \cdot \Delta\sigma_{st,max} \leq \frac{\Delta\sigma_{Rsk}}{\gamma_{Sfat}} = 0$$

Top reinforcement

Maximum steel stress range in the top reinforcement

$$\Delta\sigma_{st',max} := \max(\Delta\sigma_{st'}) = 393.122 \cdot \text{MPa}$$

Check of condition

$$\gamma_{Ffat} \cdot \Delta\sigma_{st',max} \leq \frac{\Delta\sigma_{Rsk}}{\gamma_{Sfat}} = 0$$



## D.2 Fatigue assessment of concrete in compression

### Fatigue assessment of concrete in compression, according to Eurocode 2 [EN 1992-1-1:2005 6.8.7]

The following condition should be fulfilled for sufficient fatigue resistance

$$E_{cd,max} + 0.43 \cdot \sqrt{1 - R_{eq}} \leq 1$$

$\gamma_{Cfat} := 1.5$  Coefficient taking the material uncertainties into account

$$f_{cdfat} := \frac{f_{ck}}{\gamma_{Cfat}} = 20 \cdot \text{MPa}$$

$k_4 := 3$  national parameter

$$f_{cd,c_1} := k_4 \cdot \gamma_{Cfat} \cdot f_{cdfat}$$

$k_{1c} := 1.0$  Coefficient depending on the reference number of cycles

$t_0 := 28$  Concrete age at first fatigue load application

$s_c := 0.25$  Coefficient depending on the type of cement

Coefficient for estimated concrete compressive strength at first fatigue load application

$$\beta_{cc} := e^{s_c \cdot \left(1 - \sqrt{\frac{28}{t_0}}\right)} = 1$$

Design fatigue resistance of concrete

$$f_{cd,fat_i} := k_{1c} \cdot \beta_{cc} \cdot f_{cd,c_i} = \dots \cdot \text{MPa}$$

Minimum compressive stress level in a cycle

$$i := 0 \dots (\text{length}(\sigma_{cc,min}) - 1)$$

$$E_{cd,min_i} := \frac{\sigma_{cc,min,EC2_1}}{f_{cd,fat_i}} = \dots$$

Maximum compressive stress level in a cycle

$$E_{cd,max_i} := \frac{\sigma_{cc,max_1}}{f_{cd,fat_i}} = \dots$$

Stress ratio

$$R_{eq_i} := \begin{cases} 0 & \text{if } \left( \frac{E_{cd.min}}{E_{cd.max}} \right)_i < 0 \\ 0 & \text{if } \left( \frac{E_{cd.min}}{E_{cd.max}} \right)_i \geq 1 \\ \left( \frac{E_{cd.min}}{E_{cd.max}} \right)_i & \text{otherwise} \end{cases}$$

$$\log N_{cc_i} := \begin{cases} \left( 14 \cdot \frac{1 - E_{cd.max_i}}{\sqrt{1 - R_{eq_i}}} \right) & \text{if } 14 \cdot \frac{1 - E_{cd.max_i}}{\sqrt{1 - R_{eq_i}}} \leq 307 \\ 307 & \text{otherwise} \end{cases}$$

$$N_{cc_i} := 10^{\log N_{cc_i}} \quad \text{Number of cycles until fatigue failure [EN 1992-2:2005]}$$

$$D_{cc.EC2} := \sum_{i=0}^j \frac{n_{over_i}}{\binom{\log N_{cc_i}}{10}}$$

$$D_{cc.EC2} = 0$$

$$E_{cd.max_{80}} + 0.43 \cdot \sqrt{1 - R_{eq_{80}}} = 0.69$$

$$check_i := E_{cd.max_i} + 0.43 \cdot \sqrt{1 - R_{eq_i}} \leq 1 = \dots$$

$$\sum_i check_i = 85$$

$$\sum_i check_i = \text{length}(M_{fat.max}) = 1$$

**Fatigue assessment of concrete in compression, according to Eurocode 2  
[EN 1992-1-1:2005 6.8.7]**

$$\text{check}_i := \frac{\sigma_{cc,max_i}}{f_{cd,fat_i}} \leq 0.5 + 0.45 \cdot \left( \frac{\sigma_{cc,min_i}}{f_{cd,fat_i}} \right) \leq 0.9$$

$$\sum_i \text{check}_i = 85$$

$$\sum_i \text{check}_i = \text{length}(M_{fat,max}) = 1$$

**Fatigue assessment of concrete in compression, according to fib Model  
Code 2010**

$\beta_{csus} := 0.85$  Coefficient that takes the effect of high mean stresses during loading into account. Can be assumed to 0.85 for fatigue loading.

$$f_{ctk0} := 10 \cdot \text{MPa}$$

Fatigue reference compressive strength

$$f_{ck,c_i} := k_4 \cdot v \cdot f_{ck} = \dots \cdot \text{MPa}$$

$$f_{ckfat_i} := \beta_{cc} \cdot \beta_{csus} \cdot f_{ck,c_i} = \dots \cdot \text{MPa}$$

The maximum compressive stress level caused by overturning moment.

$$S_{cmax_i} := \frac{|\sigma_{cc,max_i}|}{f_{ckfat_i}}$$

The minimum compressive stress level

$$S_{cmin_i} := \frac{|\sigma_{cc,min_i}|}{f_{ckfat_i}}$$

Check if  $S_{cmin}$  is between 0 and 0.8 , otherwise  $S_{cmin}$  is put equal to 0.8.

$$0 \leq S_{cmin_i} \leq 0.8$$

$$S_{cmin_i} := \begin{cases} 0.8 & \text{if } S_{cmin_i} > 0.8 \\ S_{cmin_i} & \text{if } 0 \leq S_{cmin_i} \leq 0.8 \end{cases}$$

$$\Delta S_{c_i} := |S_{cmax_i} - S_{cmin_i}| = \dots$$

$$\log N_{1_i} := \left[ 12 + 16 \cdot S_{\text{cmin}_i} + 8 \cdot (S_{\text{cmin}_i})^2 \right] \cdot (1 - S_{\text{cmax}_i})$$

$$\log N_{2_i} := 0.2 \cdot (\log N_{1_i}) \cdot (\log N_{1_i} - 1)$$

$$\log N_{3_i} := \begin{cases} 307 & \text{if } \Delta S_{c_i} = 0 \\ \left[ \log N_{2_i} \cdot (0.3 - 0.375 \cdot S_{\text{cmin}_i}) \cdot \left( \frac{1}{\Delta S_{c_i}} \right) \right] & \text{otherwise} \end{cases}$$

$$\log N_{\text{fat}_i} := \begin{cases} \log N_{1_i} & \text{if } \log N_{1_i} \leq 6 \\ \log N_{2_i} & \text{if } \Delta S_{c_i} \geq 0.3 - 0.375 \cdot S_{\text{cmin}_i} \wedge \log N_{1_i} > 6 \\ \log N_{3_i} & \text{otherwise} \end{cases}$$

$$\log N_{\text{fat}_i} := \begin{cases} \log N_{\text{fat}_i} & \text{if } \log N_{\text{fat}_i} \leq 307 \\ 307 & \text{otherwise} \end{cases}$$

Check the damage with Palmgren-Miner summation

$$j := (\text{length}(\log N_{\text{fat}}) - 1)$$

$$D_{\text{cc}} := \sum_{i=0}^j \frac{n_{\text{over}_i}}{\binom{\log N_{\text{fat}_i}}{10}}$$

$$D_{\text{cc}} = 0$$

### D.3 Fatigue assessment of shear reinforcement

#### Fatigue assessment of shear, according to Eurocode 2.

Assume the slab is cracked and no contribution from the concrete.

$$\sigma_{\text{ss.max}_i} := \frac{V_{\text{max}_i}}{A_{\text{ss.in}}} \quad \sigma_{\text{ss.min}_i} := \frac{V_{\text{min}_i}}{A_{\text{ss.in}}}$$

#### Fatigue assessment of the shear reinforcement

##### Steel stress range in shear reinforcement

$$\Delta \sigma_{\text{ss}_i} := \sigma_{\text{ss.max}_i} - \sigma_{\text{ss.min}_i}$$

If  $\Delta \sigma_{\text{ss}}$  is negative this means that the maximum and minimum values have been taken from 2 different strut and tie models. For all these values it is true that one of them comes from model 1 and the other from model 2-4. Since model 1 is the most used model the stress range is chosen from this model in these cases.

$$\Delta\sigma_{ss_i} := \begin{cases} |\Delta\sigma_{ss_i}| & \text{if } \Delta\sigma_{ss_i} > 0 \\ \frac{V_{1.\max_i}}{A_{ss.in}} - \frac{V_{1.\min_i}}{A_{ss.in}} & \text{otherwise} \end{cases}$$

Reference value for number of cycles until fatigue failure, depending on reinforcement type  
[EN 1992-1-1:2005 table 6.3N]

Number of cycles until fatigue failure, overturning moment.

$$i := 0 \dots (\text{length}(\Delta\sigma_{ss}) - 1)$$

$$\phi_{\text{bending}} := 64 \text{ mm}$$

$$N_{\text{ref}} := 10^6$$

$$\Delta\sigma_{\text{Rsk.stirrups}} := \Delta\sigma_{\text{Rsk}} \cdot \left( 0.35 + 0.026 \cdot \frac{\phi_{\text{bending}}}{\phi_s} \right)$$

$$\Delta\sigma_{\text{Rsk.stirrups}} = 79.408 \cdot \text{MPa}$$

$$N_{ss_i} := \begin{cases} N_{\text{ref}} \cdot \left[ \frac{\left( \frac{\Delta\sigma_{\text{Rsk.stirrups}}}{\gamma_{\text{Sfat}}} \right)^{k_1}}{\gamma_{\text{Ffat}} \cdot \Delta\sigma_{ss_i}} \right] & \text{if } \gamma_{\text{Ffat}} \cdot \Delta\sigma_{ss_i} \geq \left( \frac{\Delta\sigma_{\text{Rsk.stirrups}}}{\gamma_{\text{Sfat}}} \right) \wedge \Delta\sigma_{ss_i} \neq 0 \\ N_{\text{ref}} \cdot \left[ \frac{\left( \frac{\Delta\sigma_{\text{Rsk.stirrups}}}{\gamma_{\text{Sfat}}} \right)^{k_2}}{\gamma_{\text{Ffat}} \cdot \Delta\sigma_{ss_i}} \right] & \text{if } \gamma_{\text{Ffat}} \cdot \Delta\sigma_{ss_i} < \left( \frac{\Delta\sigma_{\text{Rsk.stirrups}}}{\gamma_{\text{Sfat}}} \right) \wedge \Delta\sigma_{ss_i} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Total fatigue damage of the shear reinforcement, caused by overturning moment, according to the Palmgren-Miner rule.

$$j := (\text{length}(N_{ss}) - 1)$$

$$D_{ss} := \sum_{i=0}^j \left( \begin{cases} 0 & \text{if } N_{ss_i} = 0 \\ \frac{n_{\text{over}_1}}{N_{ss_i}} & \text{otherwise} \end{cases} \right)$$

$$D_{ss} \leq 1 = 0$$

$$D_{ss} = 8.723$$

Maximum steel stress range in the shear reinforcement

$$\Delta\sigma_{ss,max} := \max(\Delta\sigma_{ss}) = 216.246 \cdot \text{MPa}$$

Check of condition

$$\gamma_{Ffat} \cdot \Delta\sigma_{ss,max} \leq \frac{\Delta\sigma_{Rsk.stirrups}}{\gamma_{Sfat}} = 0$$